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SOME GENERALIZED THEOREMS ON p-QUASIHYPONORMAL OPERATORS FOR $0 < p < 1$

MI YOUNG LEE AND SANG HUN LEE

ABSTRACT. Let $T = U|T|$ be the polar decomposition of p-quasihyponormal for $0 < p < 1$. Then the operator $\tilde{T}_\epsilon = |T|^\epsilon U|T|^{1-\epsilon}$, $0 < \epsilon \leq \frac{1}{2}$, is $(p+\epsilon)$-quasihyponormal if $p + \epsilon < 1$ and is quasihyponormal if $p + \epsilon \geq 1$. And we will prove that every p-quasihyponormal operator is paranormal and give an example to show that the converse is not true.

1. Introduction. Let $\mathcal{H}$ be a Hilbert space and let $\mathcal{L}(\mathcal{H})$ denote the algebra of all bounded linear operators on $\mathcal{H}$. An operator $T \in \mathcal{L}(\mathcal{H})$ is said to be *p-hyponormal* if $(T^*T)^p \geq (TT^*)^p$ for $p > 0$. An operator $T \in \mathcal{L}(\mathcal{H})$ is said to be *p-quasihyponormal* if $T^*((T^*T)^p - (TT^*)^p)T \geq 0$ for $p > 0$. If $p = 1$, then T is quasihyponormal and if $p = \frac{1}{2}$, then T is semi-quasihyponormal. It is well known that a p-quasihyponormal operator is a q-quasihyponormal operator for $q \leq p$. But the converse is not true in general. Also, it is immediate that every p-hyponormal operator is p-quasihyponormal but not necessarily conversely(see [4]). Hyponormal operators and quasihyponormal operators have been studied by many authors(see [9] and [11]). The p-hyponormal operator was first introduced by A. Aluthge and he studied basic properties of p-hyponormal operators(see [1] and [2]). Recently, S. C. Arora and P. Arora [4] introduced p-quasihyponormal as a generalization of quasihyponormal, and the operator $\tilde{T} = |T|^{\frac{1}{2}} U|T|^{\frac{1}{2}}$, where $T = U|T|$ is the polar decomposition of T. And they studied some properties of p-quasihyponormal using the operator $\tilde{T} = |T|^{\frac{1}{2}} U|T|^{\frac{1}{2}}$.

For a p-quasihyponormal operator $T = U|T|$, in section 2 we will introduce the operator $\tilde{T}_\epsilon = |T|^\epsilon U|T|^{1-\epsilon}$, $0 < \epsilon \leq \frac{1}{2}$, which is $(p+\epsilon)$-quasihyponormal if $p + \epsilon < 1$ and is quasihyponormal if $p + \epsilon \geq 1$. In section 3, we will prove

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