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On branching theorem of the pair $(G_2, SU(3))$

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Dedicated to Professor Hisao Nakagawa
on his sixtieth birthday

Let G be a compact connected Lie group and K be a closed subgroup. A finite dimensional complex irreducible representation $V^G(\lambda)$ of G with highest weight λ is decomposed into a direct sum of irreducible representations $V^K(\mu)$ of K with highest weight μ;

$$V^G(\lambda) = \sum_{\mu} m(\lambda, \mu) V^K(\mu).$$

It is an important problem to study the branching multiplicity $m(\lambda, \mu)$.

In [3], F. Sato studied the stability of branching coefficient. Roughly speaking, the branching coefficient $m(\lambda, \mu)$ satisfies $m(\lambda, \mu) = m(\lambda + \lambda_0, \mu)$ if λ_0 is a spherical representation of (G, K) and λ is sufficiently large.

In [2] the author studied the branching theorem of the pair $(G_2, SO(4))$ and obtained the following stability theorem (see section 2 for the description of the fundamental weights $\{\lambda_i\}$ of G_2).

Theorem 1 (Mashimo [2]) *Let $\lambda = m_1\lambda_1 + m_2\lambda_2$ be a dominant integral weight of G_2 and $\mu = \sum_{i=1}^{3} b_i\varepsilon_i$ be a dominant integral weight of $SO(4)$. Then*

(1) *if $m_1 \geq 2b_1 + b_2 + 4$ then $m(\lambda + 2\lambda_1, \mu) = m(\lambda, \mu)$,*

(2) *if $m_2 \geq b_1 + 1$ then $m(\lambda + 2\lambda_2, \mu) = m(\lambda, \mu)$.*

The aim of this note is to calculate the branching coefficients of the pair $(G_2, SU(3))$ and to prove the "stability" of branching coefficients.