Nihonkai Math. J.
vol. 3 (1992), 177-185

CURVATURE–ADAPTED SUBMANIFOLDS

Jürgen Berndt and Lieven Vanhecke

Abstract

We show that the classification of curvature-adapted submanifolds in $\mathfrak{P}$-spaces can be reduced to that of curvature-adapted hypersurfaces by using tubes about submanifolds. Moreover, we treat the special case of non-flat complex and quaternionic space forms. This leads to a complete classification of the curvature-adapted submanifolds in quaternionic projective spaces.

1. Introduction

In this note we study a certain class of submanifolds whose extrinsic curvature is adapted in a natural way to the intrinsic curvature of the ambient Riemannian manifold. A general measure for the extrinsic curvature of a submanifold is provided by all the shape operators A_ξ with respect to normal vectors ξ. Given a normal vector ξ to a submanifold, the Jacobi operator $R_\xi := R(.,\xi)\xi$ measures the intrinsic curvature of the ambient Riemannian manifold $\bar{M}$ in the direction of ξ. Here, R denotes the Riemannian curvature tensor of $\bar{M}$. Both A_ξ and R_ξ are self-adjoint operators; their eigenvalues represent extremal curvatures, and their eigenspaces point out directions for which the curvature becomes extremal. We say that a submanifold M of a Riemannian manifold $\bar{M}$ is *curvature-adapted* if for every normal vector ξ to M, say at $p \in M$, the Jacobi operator R_ξ leaves the tangent space T_pM of M at p invariant, that is, if

$$R_\xi(T_pM) \subset T_pM, \tag{1}$$

and if there exists a basis of T_pM consisting of eigenvectors both of A_ξ and $K_\xi := R_\xi|T_pM$, that is, if

$$A_\xi \circ K_\xi = K_\xi \circ A_\xi. \tag{2}$$

1980 Mathematics Subject Classification (1985 Revision). 53C35, 53C40, 53C55.
Keywords. Curvature-adapted or compatible submanifolds. $\mathfrak{P}$-spaces, tubes and focal sets, complex and quaternionic space forms.