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The Order of Starlikeness and Convexity of Confluent Hypergeometric Functions

KHALIDA INAYAT NOOR

ABSTRACT

We determine the conditions for which confluent hypergeometric functions are convex and starlike of order α.

Key Words and Phrases: Confluent hypergeometric functions, convex, starlike of order α, convolution, linear operator, radius of convexity.
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1. INTRODUCTION

Let A denote the class of analytic functions f in the unit disk $E = \{z: |z|<1\}$ with $f(0) = 0$, $f'(0) = 1$. We denote by S the subclass of A consisting of univalent functions. A function $f \varepsilon S^*(\alpha)$ S, $0<\alpha<1$, if and only if Re $\frac{zf'(z)}{f(z)} > \alpha$, $z \varepsilon E$. We call f a starlike function of order α in E. Also, a function $f \varepsilon S$, satisfying Re $\{\frac{(zf'(z))'}{f'(z)}\} > \alpha$, $0<\alpha<1$, $z \varepsilon E$, is called a convex function of order α and we denote the class consisting of such functions as $C(\alpha)$.

It is clear that

$$f \varepsilon C(\alpha) \text{ if, and only if, } zf' \varepsilon S^*(\alpha), \tag{1.1}$$

Let c be a complex numbers with $c \neq 0, -1, -2, \ldots.$, and consider the function defined by

$$\phi(a;c;z) = {}_1F_1(a;c;z) = 1 + \frac{a}{c}\frac{z}{1!} + \frac{a(a+1)}{c(c+1)}\frac{z^2}{2!} + \ldots. \tag{1.2}$$

This function is called Confluent (or Kummer) hypergeometric function and it is analytic in C. It satisfies Kummer's hypergeometric differential equation

$$zw''(z) + (c-z)w'(z) - aw(z) = 0 \tag{1.3}$$

If we let $(d)_k = \frac{\Gamma(d+k)}{\Gamma(d)}$

$$= d(d+1)\ldots.(d+k-1),$$