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Necessary and Sufficient Conditions for Oscillation of Second Order Autonomous Neutral Equations with Distributed Delay

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Absract.

In this paper the autonomous neutral equation with distributed delay

$$\frac{d^2}{dt^2}[x(t)+\delta_1\int_0^\tau x(t-s)dr_1(s)]+\delta_2\int_0^\tau x(t-s)dr_2(s)=0,$$

where $\delta_i=\pm 1$, i=1,2, is considered. It is proved that the necessary and sufficient condition for all solutions of this equations to oscillate is that the corresponding characteristic equation

$$z^2(1+\delta_1\int_0^\tau e^{-zs}dr_1(s))+\delta_2\int_0^\tau e^{-zs}dr_2(s)=0$$

should have no real root.

1.Introduction.

To the problem of obtaining necessary and sufficient conditions for oscillation of all solutions of second and higher order neutral differential equations the papers [1]-[5] are devoted. The neutral equations considered are with a finite number of concentrated delays. The most general results were obtained in [1] and [4], in [1] systems of equations being investigated. The only result in this direction for neutral equations with distributed delay is the work [6] which concerns first order equations. In the present paper the equation

$$\frac{d^2}{dt^2}[x(t)+\delta_1\int_0^\tau x(t-s)dr_1(s)]+\delta_2\int_0^\tau x(t-s)dr_2(s)=0, \tag{1}$$

is investigated. It is proved that the necessary and sufficient condition for all solutions of (1) to oscillate is that the characteristic equation of (1)

$$Q(z)=z^2(1+\delta_1\int_0^\tau e^{-zs}dr_1(s))+\delta_2\int_0^\tau e^{-zs}dr_2(s)=0 \tag{2}$$

should have no real root. The result is a generalization of the work [3].

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