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A SUFFICIENT CONDITION FOR A GRAPH TO BE TRACEABLE *

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ABSTRACT

We prove that a 2-connected graph of order p is traceable if for all distinct vertices u and v, dist(u,v)=2 implies that $|N(u) \cup N(v)| \geqslant (p-1)/2$. This result was once conjectured by T. E. Lindquester.

INTRODUCTION

A path in a graph G is called a hamiltonian path in G if it contains all the vertices of G. A graph is traceable if it has a hamiltonian path. The neighborhood of a vertex v, denoted N(v), is the set of all vertices adjacent to v. We define the distance, denoted dist(u,v), between two vertices u and v as the minimum of the lengths of all u-v paths. Let $NC2=\min|N(u) \cup N(v)|$, where the minimum is taken over all pairs of vertices u,v that are at distance two in the graph. Refer to [2] for other terminology.

T.E.Lindquester has given the following theorem in [1]:

Theorem 1. Let G be a 2-connected graph of order p. If

$$NC2 > (2p-5)/3 ,$$

then G is traceable.

He also raised the following conjecture:

Conjecture. Let G be a 2-connected graph of order p. If

$$NC2 \geqslant (p-1)/2 ,$$

then G is traceable.

He also pointed out that the 2-connected bipartite graph $K(n-2,n)$, $n \geqslant 4$, is nontraceable. If $|K(n-2,n)|=p$, then $NC2 \geqslant (p-2)/2$. Thus, the conjecture is the best possible result of this nature that can be obtained.

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