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ON THE ALMOST EVERYWHERE CONVERGENCE OF BOCHNER-RIESZ MEANS OF MULTIPLE FOURIER INTEGRALS FOR RADIAL FUNCTIONS

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ABSTRACT. Let $n \geqq 2$ and $(S_*^\delta f)(x) = \sup_{R>0} |(S_R^\delta f)(x)|$, where $S_R^\delta f$ is the Bochner-Riesz mean of order δ of the Fourier integral for f on R^n. We show that the operator S_*^δ is bounded from the Lorentz space $L^{p,1}(R^n)$ into $L^{p,\infty}(R^n)$ on the critical line $\delta = n(1/p-1/2)-1/2$ for $2n/(n+2) \leqq p \leqq 2n/(n+1)$ besides $p>1$ when acting on radial functions.

§1. Introduction.

Let R^n be the $n(\geqq 2)$-dimensional Euclidean space and for any $x=(x_1,\ldots,x_n)$, $y=(y_1,\ldots,y_n)$ in R^n, we denote $(x,y)=x_1y_1+\cdots+x_ny_n$ and $|x|=(x,x)^{1/2}$.

For the Fourier integral of a function $f \in L^p(R^n)$ $(1 \leqq p \leqq 2)$, its Bochner-Riesz mean of order $\delta \geqq 0$ is defined by

$$(1) \quad (S_R^\delta f)(x) = (\sqrt{2\pi})^{-n} \int_{|y|<R} \left(1-\frac{|y|^2}{R^2}\right)^\delta \hat{f}(y)\, e^{i(x,y)} dy,$$

where $\hat{f}(y)$ is the Fourier transform of f, i.e.