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HYPERSURFACES OF THE TWO-DIMENSIONAL COMPLEX PROJECTIVE SPACE

Sharief Deshmukh and M.A. Al-Gwaiz

Abstract : We consider a compact, simply connected, real hypersurface of the two dimensional complex projective space CP^2 of constant holomorphic sectional curvature 4, and obtain a criterion for the hypersurface to be diffeomorphic to S^3 in terms of an inequality which relates the length of the second fundamental form and the mean curvature.

1. Introduction

Real hypersurfaces of the complex projective space CP^n have not been studied as extensively as those of real space forms. One of the reasons for this is that the Codazzi equation is simpler for hypersurfaces in real space forms. To overcome this deficiency, Lawson [2] considered the Riemannian submersion $\pi: S^{2n+1} \to CP^n$ which gives rise,for a real hypersurface M in CP^n, to a hypersurface $\pi^{-1}(M)$ in S^{2n+1} such that $\pi: \pi^{-1}(M) \to M$ is a Riemannian submersion with totally geodesic fibers. The properties of a hypersurface $\tilde{M}$ of S^{2n+1}, which are invariant under free S^1 action, can then be projected on the hypersurfaces of CP^n provided that the corresponding S^1 action on S^{2n+1} induces a free S^1 action on $\tilde{M}$. Lawson [2] used this idea to study the minimal hypersurfaces of CP^n, and Okumura [4] to study those