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Quinary code construction of the Leech lattice

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1 Introduction

Let Γ_{24} be the genus consisiting of all the equivalence classes of positive definite even unimodular lattices of rank 24. Let $\mathbf{L}$ be an element of Γ_{24}. An element $\mathbf{x}$ in $\mathbf{L}$ is called a 2m-vector if $\mathbf{x}$ satisfies $(\mathbf{x},\mathbf{x}) = 2m$ for some positive integer m. In Γ_{24} , there is a unique class $\{\ \mathbf{L}\ \}$, in which there is no 2-vector. Such a lattice is first given by Leech [3], and it is called the Leech lattice. Leech's construction is based on the binary Golay code of length 24. Later on Leech and Sloane [4] gave another construction for the Leech lattice based on the self-dual ternary codes of length 24. After that various constructions for the Leech lattice have been given (e.g. [6], [16]) because of the plentiful structures and properties which the Leech lattice has.
In [10], [11] and [12] we have investigated the notion of the c-sublattice $\mathbf{M}$ of an even unimodular lattice $\mathbf{L}$. As a byproduct we give a construction of the Leech lattice by using a self-dual code $\mathbf{C}$ over GF(5) of length 24.

Remark 1 *We expect that the present method may serve to search positive definite even unimodular extremal lattices of ranks 72, 80 or 88. However it would take much labors, such as giving codes over GF(5) with large minimum distances or computing Lee weight enumerators of such codes, to attack the problem.*

Remark 2 *It seems to us that there would be another constructions of the Leech lattice using (special) self-dual codes of length 24 over GF(p), the field*