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An example of a totally geodesic foliation which is perpendicular to a certain non-singular Killing field on an arbitrary three-dimensional Lorentzian lens space

Ken Yokumoto

Abstract

We construct a totally geodesic foliation which is perpendicular to a certain non-singular Killing field on an arbitrary three-dimensional Lorentzian lens space.

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1 Introduction

Totally geodesic foliations on Lorentzian manifolds are studied by several authors ([BMT], [CR], [M], [Y1], [Y2], [Y3], [Z2], [Z3], [Z4]).

An example of a codimension-1 totally geodesic foliation containing spacelike, timelike, and lightlike leaves appeared first in [Y1], and it was obtained as $\ker g(X,\cdot)$, where X is a non-singular Killing field for a Lorentzian metric g on the 2-torus T^2. So it seemed a "typical" example of a codimension-1 totally geodesic foliation. These typical examples, i.e., codimension-1 totally geodesic foliations perpendicular to non-singular Killing fields, were treated and classified in [Y3].

In [Y2], we constructed Lorentzian geodesible foliations of closed 3-manifolds having Heegaard splittings of genus one, i.e., lens spaces $L(p,q)$ of type (p,q), the 3-sphere $S^3 \cong L(1,0)$, and $S^2 \times S^1 \cong L(0,1)$. Here a Lorentzian geodesible foliation means a totally geodesic foliation for some, in general incomplete, Lorentzian metric. However, the constructed example of a totally geodesic foliation $\mathcal{F}$ was not a typical example, that is, $\mathcal{F}$ was not obtained as $\ker g(X,\cdot)$ for some non-singular Killing field X. So the natural question concerning the existence problem of typical examples arises. More precisely, we have

Question 1 *Can we give a non-singular Killing field X for some Lorentzian metric of a 3-manifold such that the distribution $\ker g(X,\cdot)$ is completely integrable?*