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NON-SMOOTH GALOIS POINT ON A QUINTIC CURVE WITH ONE SINGULAR POINT

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ABSTRACT. Let C be an irreducible plane quintic curve with only one singular point P, which is a double point. Then, we consider a projection of C from P. This projection induces an extension of rational function fields $k(C)/k(\mathbb{P}^1)$. In this paper, we give the defining equation of the curve C when the extension is Galois.

1. INTRODUCTION

Let k be an algebraically closed field of characteristic zero, which we fix as the ground field of our discussion. Let C be an irreducible (possibly singular) curve of degree d in the projective plane $\mathbb{P}^2 = \mathbb{P}^2(k)$ and $K = k(C)$ the rational function field of C. For each point $P \in C$, let $\pi_P : C \cdots \to l$ be a projection from C to a line l with the center P. This rational map induces the extension of fields $K/k(l)$. The structure of this extension does not depend on the choice of l, but on P, so that we write K_P instead of $k(l)$.

Definition 1. *A point $P \in C$ is called a Galois point if the extension K/K_P is Galois. In particular, a Galois point is called a non-smooth Galois point [resp. a smooth Galois point] if it is singular. [resp. nonsingular.]*

In the papers [5], [6] and [8], Yoshihara raised the following questions:

(1) When is the extension K/K_P Galois? Namely, when is the point P Galois?
(2) How many Galois points do there exist on C (or $\mathbb{P}^2\setminus C$)?
(3) Let L_P be the Galois closure of K/K_P. What can we say about L_P?
(4) What is the Galois group $\mathrm{Gal}(L_P/K_P)$?
(5) Determine intermediate fields between K_P and L_P.

These were treated in detail for nonsingular plane curves in papers [5], [6], [8] and Miura's paper [2]. Miura also studied these questions for singular plane quartic curves in [1] and [3].

Let $(X : Y : Z)$ be homogeneous coordinates on $\mathbb{P}^2$ and (x, y) affine coordinates such that $x = X/Z$ and $y = Y/Z$. For a nonsingular plane curve, we have an answer to Question (1) as follows.

Proposition 1 ([8], Proposition 5). *Let C be a nonsingular plane curve of degree d $(d \geq 4)$. Then, the point $P \in C$ is Galois if and only if the defining equation*

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