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# Weighted composition operators between weighted Bergman spaces in the unit ball of $\mathbb{C}^n$

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**Abstract**

Let $\varphi$ be a holomorphic self-map of the unit ball $B$ in $\mathbb{C}^n$ and $\psi$ a holomorphic function in $B$. Let $A^p(\nu_\alpha)$ denote the weighted Bergman space in $B$. In this paper, we characterize the boundedness and the compactness of the weighted composition operator $W_{\varphi,\psi} : f \mapsto \psi(f \circ \varphi)$ from $A^p(\nu_\alpha)$ into $A^q(\nu_\beta)$ $(0 < p \leq q < \infty, -1 \leq \alpha, \beta < \infty)$, in terms of the Carleson-type measures. We also consider the boundedness and the compactness of $W_{\varphi,\psi} : A^p(\nu_\alpha) \to H^\infty(B)$, the space of the bounded holomorphic functions in $B$.

## 1 Introduction

Throughout this paper, let $n$ be a fixed integer. Let $B$ and $S$ denote the unit ball and the unit sphere of the complex $n$-dimensional Euclidean space $\mathbb{C}^n$, respectively. Let $\nu$ and $\sigma$ denote the normalized Lebesgue measures on $B$ and $S$, respectively. For each $\alpha \in (-1, \infty)$, we set $c_\alpha = \Gamma(n+\alpha+1)/\{\Gamma(n+1)\Gamma(\alpha+1)\}$ and $d\nu_\alpha(z) = c_\alpha(1-|z|^2)^\alpha d\nu(z)$ $(z \in B)$. Note that $\nu_\alpha(B) = 1$. Let $H(B)$ denote the space of all holomorphic functions in $B$. For each $p \in (0, \infty)$ and $\alpha \in (-1, \infty)$, the *weighted Bergman space* $A^p(\nu_\alpha)$ and the *Hardy space* $H^p(B)$ are defined by

$$A^p(\nu_\alpha) = \left\{ f \in H(B) : \|f\|^p_{A^p(\nu_\alpha)} \equiv \int_B |f|^p d\nu_\alpha < \infty \right\},$$

$$H^p(B) = \left\{ f \in H(B) : \|f\|^p_{H^p} \equiv \sup_{0<r<1} \int_S |f_r|^p d\sigma < \infty \right\},$$

where $f_r(z) = f(rz)$ for $r \in (0,1)$, $z \in \mathbb{C}^n$ with $rz \in B$. For convenience' sake, the spaces $H^p(B)$ are denoted by the symbols $A^p(\nu_{-1})$ $(0 < p < \infty)$. Note that $\lim_{\alpha \downarrow -1} \|f\|_{A^p(\nu_\alpha)} = \|f\|_{H^p}$ for $p \in (0, \infty)$ and $f \in H(B)$. (See [1, §0.3 and p.25].)

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