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On Four-dimensional Generalized Complex Space Forms

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Abstract

F. Tricerri and L. Vanhecke [8] proved that a $2n$ $(n \geq 3)$-dimensional generalized complex space is a real space form or a complex space form. In this note, we shall show that this result is extendable to 4-dimensional case.

1 Introduction

Let (V, g) be an n-dimensional real vector space with positive definite inner product g and denote by $\mathcal{R}(V)$ the subspace of $V^* \otimes V^* \otimes V^* \otimes V^*$ consisting of all tensors having the same symmetries as the curvature tensor of a Riemannian manifold, including the first Bianchi identity. F. Tricerri and L. Vanhecke [8] gave the complete and irreducible decomposition of $\mathcal{R}(V)$ under the action of $\mathcal{U}(n)$. They then applied these algebraic results to the curvature tensors of almost Hermitian manifolds.

A $2n$ $(n \geq 2)$ - dimensional almost Hermitian manifold $M = (M, J, g)$ is called a *generalized complex space form* if the curvature tensor R takes the following form:

$$(1.1) \qquad \begin{aligned} R &= \frac{\tau + 3\tau^*}{16n(n+1)}(\pi_1 + \pi_2) + \frac{\tau - \tau^*}{16n(n-1)}(3\pi_1 - \pi_2) \\ &= \frac{(2n+1)\tau - 3\tau^*}{8n(n-1)(n+1)}\pi_1 + \frac{(2n-1)\tau^* - \tau}{8n(n-1)(n+1)}\pi_2 \end{aligned}$$

for some smooth functions τ and τ^* and here

$$\pi_1(x, y)z = g(y, z)x - g(x, z)y$$

and

$$\pi_2(x, y)z = g(Jy, z)Jx - g(Jx, z)Jy - 2g(Jx, y)Jz$$

for all x, y, $z \in T_pM$, $p \in M$.

The concept of generalized complex space form is a natural generalization of a complex space form (i.e. Kähler manifold of constant holomorphic sectional curvature) which has been introduced by F. Tricerri and L. Vanhecke [8]. They showed