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SOME LINEAR FUNCTIONAL AND FOURIER TRANSFORM OVER $\mathcal{K}'_{e,k}$

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ABSTRACT. We introduce the space $\mathcal{K}_{e,k}$ that is the vector space of all C^∞ - functions f such that $exp(e^{k|x|})\partial^\alpha f$ vanishes at infinity for all $\alpha \in N^n, k \in Z, k < 0$ and its dual $\mathcal{K}'_{e,k}$. For $f, g \in \mathcal{K}'_{e,k}$, we study the linear functional $f \circledast g$ on $\mathcal{K}_{e,k}$ defined by

$$< f \circledast g > = < f(x), < g(y), \phi(x+y) >>, \quad \phi \in \mathcal{K}_{e,k}.$$

Also, we show a representation theorem for the usual distributional Fourier transform over the spaces $\mathcal{K}'_{e,k}$, and an inversion formula which enables to prove that $\mathcal{K}'_{e,k}$ is a commutative algebra with unit element with respect to $\circledast$

1. Introduction

The Schwartz space $\mathcal{S}$ is the space of all infinitely differentiable function f on R^n such that $(1+|x|^2)^k \partial^\alpha f(x)$ vanishes at infinity for all $k \in Z$ and all $\alpha \in N^n$. The space $\mathcal{S}$ is equipped with the locally convex topology defined by the family $(q_{k,\alpha})$ of seminorms $(q_{k,\alpha}) = (1+|x|^2)^k |\partial^\alpha f(x)|$, where k runs through N and α through N^n. By $\mathcal{S}'$, we mean the space of continuous linear functionals on $\mathcal{S}$. Motivated by the Schwartz space $\mathcal{S}$, J. Horváth introduced the space $\mathcal{S}_k$, k is a fixed integer, that is defined as the vector space of all functions f on R^n such that $(1+|x|^2)^k \partial^\alpha f(x)$ vanishes at infinity for all $\alpha \in N^n$ in [3]. Horváth defined on $\mathcal{S}_k$ the seminorms $(\mu_{k,\alpha}) = (1+|x|^2)^k |\partial^\alpha f(x)|$ for a fixed k and every $\alpha \in N^n$. And B.J.Gonzalez and E.R.Negrin studied the convolution and Fourier transform over $\mathcal{S}_k, k \in Z, k < 0$, in [1] and [2], respectively.

In the meantime, the Schwartz space $\mathcal{S}$ is extended by G. Sampson and Z. Zielezny in [5]. They introduced the space $\mathcal{K}_p, p > 1$, of the space of all infinitely differentiable functions f on R^n such that $e^{k|x|^p} \partial^\alpha f(x)$ vanishes at infinity for all $k \in Z$ and all $\alpha \in N^n$. The space $\mathcal{K}_p, p > 1$, is equipped with the locally convex topology defined by the family of seminorms $(\gamma_{k,\alpha}) = e^{k|x|^p} |\partial^\alpha f(x)|$, where k runs

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