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K-THEORY OF CONTINUOUS FIELDS OF QUANTUM TORI

TAKAHIRO SUDO

ABSTRACT. In this paper we study K-theory of continuous fields of quantum tori. For this purpose we review and compute K-theory of C^*-algebras of continuous functions on the tori and that of the quantum (or noncommutative) tori by obtaining the formulas for counting generators of their K-groups.

0. INTRODUCTION

Our first motivation for this study is the following:

Problem. *Let $\Gamma(X, \{\mathfrak{A}_t\}_{t\in X})$ be the C^*-algebra of a continuous filed on a locally compact Hausdorff space X with the fibers $\mathfrak{A}_t$. Then how does one compute its K-groups in terms of the base space X and the fibers $\mathfrak{A}_t$?*

See Fell [F] and Dixmier [Dx] for the theory of continuous fields of C^*-algebras. As a step toward solving the problem, we focus our attention to the case where $X = \mathbb{T}^n$ the tori and $\mathfrak{A}_t = \mathfrak{A}_\Theta$ the quantum (or noncommutative) tori. Fortunately, the K-groups of the C^*-algebras $C(\mathbb{T}^n)$ of continuous functions on $\mathbb{T}^n$ as well as $\mathfrak{A}_\Theta$ are well known. Also, the Bott generator for the K_0-group of $C(\mathbb{T}^2)$ and the Rieffel projections for the K_0-group of the quantum 2-tori are well known. However, the generators of the K-groups of $C(\mathbb{T}^n)$ and the quantum n-tori for $n \geq 3$ seem to be little well known in the literature. Therefore, in Section 1 we review and study the K-groups of $C(\mathbb{T}^n)$ by obtaining the formulas for counting generators given by the generalized Bott projections. In Section 2 we review and study the K-groups of $\mathfrak{A}_\Theta$ by obtaining the formulas for counting generators given by the generalized Rieffel projections. Using these explicit formulas for counting generators of the K-groups, in Section 3 we obtain a partial answer to the Problem.

Notation. Let $C(X)$ be the C^*-algebra of all continuous complex-valued functions on a compact Hausdorff space X. Let $K_*(\mathfrak{A})$ for $* = 0, 1$ be the K-groups of a C^*-algebra $\mathfrak{A}$. See [Bl], [RLL], [Wo] for details about the K-theory of C^*-algebras.

1. THE C^*-ALGEBRAS OF CONTINUOUS FUNCTIONS ON THE TORI

In this section we first briefly recall the K-theory of the C^*-algebras of continuous complex-valued functions on the tori.

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