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GROWTH SEQUENCES FOR FLAT DIFFEOMORPHISMS OF THE INTERVAL

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1. Introduction and statement of results

Let f be a C^1-diffeomorphism of the interval $[0;1]$. We define a *growth sequence* for f by

$$\Gamma_n(f) = \exp\|\log Df^n\| = \max(\|Df^n\|, \|Df^{-n}\|),$$

where f^n is nth iteration of f and $\|Df^n\| = \max_{x\in[0;1]} |Df^n(x)|$.

Let $\mathrm{Fix}(f)$ be the set of fixed points of f. In the case f is of class C^r, for $x \in \mathrm{Fix}(f)$ x is called *r-flat* if $Df(x) = 1$ and $D^n f(x) = 0$ for $2 \leq n \leq [r]$. f is called *r-flat* if every $x \in \mathrm{Fix}(x)$ is r-flat.

In this paper, we answer the question raised in the paper by L. Polterovich and M. Sodin [2]. We show :

Theorem 1. *Let f be a 2-flat diffeomorphism of the interval. Then,*

$$\lim_{n\to\infty} \frac{\Gamma_n(f)}{n^2} = 0.$$

Theorem 2. *There exists an ∞-flat diffeomorphism f of the interval such that for every $\alpha < 2$,*

$$\limsup_{n\to\infty} \frac{\Gamma_n(f)}{n^\alpha} = \infty.$$

Independently, A. Borichev shows similar results [1].

2. Proof of Theorem 1

The argument in Proof of Theorem 1 is a slight modification of its in [2]. The following is useful.

Lemma 3. (Denjoy) *Let f be a C^2-diffeommorphism of $[0;1]$. If $J \in [0;1]$ is a closed interval such that $\mathrm{Int}(J) \cap f(\mathrm{Int}(J)) = \emptyset$ then there exists a positive constant C depending on f such that for every $n \in \mathbb{N}$ and every $x, y \in J$*

$$\frac{1}{C} \leq \frac{Df^n(x)}{Df^n(y)} \leq C.$$