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THE SOLVABLE STRUCTURE OF THE C^*-ALGEBRAS OF CERTAIN SUCCESSIVE SEMI-DIRECT PRODUCTS

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ABSTRACT. As the main results we construct finite composition series of group C^*-algebras of certain successive semi-direct products of $\mathbb{C}^n$ by $\mathbb{R}$, $\mathbb{T}$ or $\mathbb{Z}$ such that their subquotients are tensor products involving commutative C^*-algebras, non-commutative tori and the C^*-algebra of all compact operators. As an application, we estimate the stable rank and connected stable rank of the group C^*-algebras of these connected or disconnected solvable Lie groups. Also, we introduce a class of C^*-algebras that are C^*-solvable in some sense.

0. INTRODUCTION

First recall that any simply connected, solvable Lie group G is isomorphic to a successive semi-direct product by $\mathbb{R}$ as follows:

$$G \cong (\cdots((\mathbb{R}^n \rtimes_{\alpha^1} \mathbb{R}) \rtimes_{\alpha^2} \mathbb{R})\cdots) \rtimes_{\alpha^m} \mathbb{R}$$

for $n \geq 1$ and $m \geq 0$ and α^j $(1 \leq j \leq m)$ actions by $\mathbb{R}$ (cf.[OV, Section 3 in Chapter 2]). It has been an interesting and important problem to study the (algebraic) structure of group C^*-algebras for all or certain G. Some remarkable results related with this problem were obtained by Green [Gr1] [Gr2] (for the imprimitivity theorem for crossed products and simple quotients of group C^*-algebras), Poguntke [Pg] (for simple quotients of group C^*-algebras of connected Lie groups) and Rosenberg [Rs] (for group C^*-algebras of certain solvable Lie groups with lower dimensions). On the other hand, we have explicitly studied the structure of group C^*-algebras in the case of semi-direct products of $\mathbb{C}^n$ by $\mathbb{R}$, $\mathbb{T}$ or $\mathbb{Z}$ (see [Sd5], [Sd10], [Sd6] respectively) and in the case of semi-direct products of $\mathbb{C}^n$ by $\mathbb{R}^n$ or $\mathbb{Z}^n$ with the diagonal actions (see [Sd9], [Sd8] respectively). In this paper we consider the structure of group C^*-algebras of some successive semi-direct products of $\mathbb{C}^n$ by $\mathbb{R}$, $\mathbb{T}$ or $\mathbb{Z}$ by using some results of [Sd5], [Sd10], [Sd6] inductively (cf. [Sd7], [Sd10] for group C^*-algebras of certain semi-direct products by the generalized discrete Heisenberg groups or Heisenberg Lie groups respectively). This attempt for the problem is still far from the general case, but should be a steady step and

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