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Two classes of Lorentzian stationary surfaces in semi-Riemannian space forms

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Abstract. We give certain two classes of 2-dimensional Lorentzian metrics which can be realized as induced metrics of Lorentzian stationary surfaces in semi-Riemannian space forms.

1. Introduction

Let $N_\nu^n(c)$ denote the n-dimensional simply connected semi-Riemannian space form of constant curvature c and index ν. A surface in $N_\nu^n(c)$ is called Lorentzian if its induced metric is Lorentzian. We say that a Lorentzian surface in $N_\nu^n(c)$ is stationary if its mean curvature vector vanishes identically. We are interested in the following question: Which 2-dimensional Lorentzian metrics can be realized as induced metrics of Lorentzian stationary surfaces in $N_\nu^n(c)$?

There are several related results for minimal surfaces in Riemannian space forms (cf. [4], [5], [2], [3]). In the previous paper [7], refering to [3], we gave two classes of 2-dimensional Riemannian metrics which can be realized as spacelike stationary surfaces in $N_\nu^n(c)$. In this paper, we will give two classes of 2-dimensional Lorentzian metrics which can be realized as Lorentzian stationary surfaces in $N_\nu^n(c)$.

Let M be a 2-dimensional Lorentzian manifold with Gaussian curvature K and Laplacian Δ. For each real number c, set

$$F_1^c = 2(K-c), \quad F_{p+1}^c = F_p^c + 2(p+1)K - \sum_{q=1}^{p} \Delta \log(F_q^c) \ \text{ if } \ F_p^c > 0.$$

Our results are stated as follows.

Theorem 1. *Let M be a 2-dimensional simply connected Lorentzian manifold. Suppose that $F_p^c > 0$ for $p < m$, and $F_m^c = 0$ identically. Then there*