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SCHOLZ ADMISSIBLE MODULI OF FINITE GALOIS EXTENSIONS OF ALGEBRAIC NUMBER FIELDS

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ABSTRACT. Let K be a finite Galois extension over an algebraic number field k with Galois group G. We call a modulus $\mathfrak{M}$ of K Scholz admissible when the Schur multiplier of G is isomorphic to the number knot of K/k modulo $\mathfrak{M}$. This paper develops a systematic treatment for Scholz admissibility. We first reduce the problem to the local case, in particular, to the strongly ramified case, and study this case in detail. A main object of local Scholz admissibility is $H^{-1}(G, U_K^{(s)})$ in the strongly ramified case. In the case where K/k is totally strongly ramified of prime power degree p^n, we prove that the natural homomorphism : $H^{-1}(G, U_K^{(r+s)}) \to H^{-1}(G, U_K^{(s)})$ is trivial for $s \geq 1$, where r denotes the last ramification number. This result describes a basic situation for vanishing of $H^{-1}(G, U_K^{(s)})$. Using this result for a Galois tower $K \supset L \supset k$ with a totally strongly ramified cyclic extension L/k we prove a relationship between Scholz admissible moduli of K/L and K/k. This gives a way to estimate for Scholz conductor of K/k from the ramification in K/k. As an application of this result we give an alternative proof of a result of Fröhlich.

1. INTRODUCTION

Let k be an algebraic number field of finite degree. Let K be a finite Galois extension over k with Galois group $G = Gal(K/k)$. Let $\mathfrak{M}$ be a Galois modulus of K/k, i.e., a finite product of primes of K which satisfies $\mathfrak{M}^\sigma = \mathfrak{M}$ for any $\sigma \in G$. It is known that the number knot of K/k modulo $\mathfrak{M}$ is an epimorphic image of Schur multiplier $H^{-3}(G, \mathbb{Z})$, i.e., there exists a natural epimorphism

$$H^{-3}(G, \mathbb{Z}) \to k \cap N_{K/k}(J_{K\mathfrak{M}})/N_{K/k}(K_{(\mathfrak{M})}) \tag{1.1}$$

(cf. Proposition 2.7). A. Scholz [9] developed the knot theory in relation to the Hasse norm principle, and established fundamental properties of this type without using cohomology. About forty years after, W. Jehne [6] gave a reformation and a generalization of Scholz's knot theory with new proofs using cohomology. Following Jehne, P. Heider [4] studied the knot theory modulo $\mathfrak{M}$. In his paper [4], Heider called $\mathfrak{M}$ a Scholz conductor when (1.1) is an isomorphism and studied some properties of Scholz conductors. In particular, he proved that a sufficiently large $\mathfrak{M}$ is a Scholz conductor of K/k. Before Heider's work, Shirai [8] determined such moduli of K/k in the case where K/k is tamely ramified. In the general case, however, it is not easy to determine such moduli explicitly.

In this paper we call $\mathfrak{M}$ Scholz admissible when (1.1) is an isomorphism with slightly modified terminology. The purpose of this paper is to develop a systematic treatment of Scholz admissibility.