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On extension of representations of $so(n+1,1)$ to representations of $so(n+1,2)$

Kiyotaka II

Abstract

In the present paper, we construct representations of the Lie algebra $so(n+1,2)$ on $C^\infty(S^n)$ by extending a representation of the Lie algebra $so(n+1,1)$ on $C^\infty(S^n)$ which arises from the action of Lorentz group $SO(n+1,1)$ on S^n as conformal transformations.

1 Introduction and statement of the result

Let $\mathbf{R}^{n+1}$ be $(n+1)$-dimensional Euclidean space with cartesian coordinates $x_1,\ldots,x_{n+1}$. For $x=(x_1,\ldots,x_{n+1})\in\mathbf{R}^{n+1}$, the norm of x is defined by $\|x\|=\sqrt{(x_1)^2+\cdots+(x_{n+1})^2}$. Let $S^n=\{x\in\mathbf{R}^{n+1}\mid \|x\|=1\}$ be the unit sphere in $\mathbf{R}^{n+1}$. Let $C^\infty(S^n)$ denote the linear space of complex-valued C^∞ functions on S^n. The special orthogonal group $SO(n+1)$ acts on S^n as an isometry group. This action induces a representation of the Lie algebra $so(n+1)$ on $C^\infty(S^n)$. Let $SO(n+1,1)$ denote the Lorentz group with its Lie algebra $so(n+1,1)$. It is well-known that the action of $SO(n+1)$ on S^n can be extended to an action of $SO(n+1,1)$ on S^n. This action induces an irreducible representation of the Lie algebra $so(n+1,1)$ on $C^\infty(S^n)$. We denote this representation by $\mathrm{Rep}_0(so(n+1,1))$.

Let us now consider the Lie algebra $so(n+1,2)$ of the Lie group $SO(n+1,2)$. Let E_{ij} denote the $(n+3)\times(n+3)$ matrix with the (i,j)-component 1 and the others are 0. The following are basis of $so(n+1,2)$.

$$E_{ij}-E_{ji}\quad(1\le i<j\le n+1),$$
$$E_{j,n+2}+E_{n+2,j}\quad(1\le j\le n+1),$$
$$E_{j,n+3}+E_{n+3,j}\quad(1\le j\le n+1),$$
$$E_{n+2,n+3}-E_{n+3,n+2}.$$