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JACOBI OPERATORS ON A SEMI-INVARIANT SUBMANIFOLD OF CODIMENSION 3 IN A COMPLEX PROJECTIVE SPACE

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ABSTRACT. In this paper, we characterize some semi-invariant submanifolds of codimension 3 in a complex projective space $\mathbb{C}P^{n+1}$ in terms of the shape operator A, the structure tensor field ϕ and the Jacobi operator R_ξ with respect to the structure vector field ξ.

0. Introduction

A submanifold M is called a CR submanifold of a Kaehlerian manifold $\tilde{M}$ with complex structure J if it is endowed with a pair of mutually orthogonal and complementary differentiable distribution $(T, T^\perp)$ such that T is J-invariant, and $T^\perp$ is totally real ([1], [19]). In particular, M is said to be a *semi-invariant submanifold* if $dim T^\perp = 1$, and the unit normal in $JT^\perp$ is called a *distinguished normal* to M ([2], [17]). In this case, M admits an induced almost contact metric structure (ϕ, ξ, g).

A typical example of a semi-invariant submanifold is real hypersurfaces. Takagi([15]) classified homogeneous real hypersurfaces of a complex projective space by means of six model spaces of type A_1, A_2, B, C, D and E, further he explicitly write down their principal curvatures and multiplicities in the table in [16].

Cecil and Ryan [3] extensively investigated a real hypersurface which is realized a tube of constant radius r over a complex submanifold of $\mathbb{C}P^n$ on which ξ is principal curvature vector with principal curvature $\alpha = 2\cot 2r(A\xi = \alpha\xi)$ and the corresponding focal map φ_r has constant rank, where we denote by A the shape operator of a real hypersurface in $\mathbb{C}P^n$.

On the other hand, Okumura [10] characterized real hypersurfaces of type A_1 and A_2 by the property that the shape operator A and structure tensor field ϕ commute. Namely he proved

Theorem O [10]. *Let M be a connected real hypersurface of $\mathbb{C}P^n$. If M satisfies $\phi A = A\phi$, then M is locally congruent to one of the following spaces:*

(A_1) *a geodesic hypersphere (that is, a tube of radius r over a hyperplane $\mathbb{C}P^{n-1}$, where $0 < r < \frac{\pi}{2}$),*
(A_2) *a tube of radius r over a totally geodesic $\mathbb{C}P^k (1 \leq k \leq n-2)$, where $0 < r < \frac{\pi}{2}$.*

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