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# Nonlinear perturbations of a class of integrated semigroups on non-convex domains

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ABSTRACT. Nonlinear continuous perturbations of integrated semigroups are treated from the point of view of the theory of semigroups of nonlinear operators on nonconvex domains. Given an integrated semigroup $W(t)$ with generator $A$ in a Banach space $X$, a general class of nonlinear perturbations on nonconvex domains is introduced by means of a lower semicontinuous functional $\varphi$. Generation and characterization of nonlinear semigroups are discussed in terms of semilinear stability condition and subtangential condition. The local Lipschitz continuity and growth condition for the nonlinear semigroups are restricted by $\varphi$ on a Banach space $X$ under consideration. In the case where both $\varphi$ and the domains of perturbing operators are convex, a Hille-Yosida type theorem is obtained.

## 1. Introduction

Of concern in this paper are the semilinear problems in a Banach space $(X, |\cdot|)$ of the form

$$\text{(SP)} \qquad u'(t) = (A + B)u(t); \quad t > 0; \qquad u(0) = v.$$

Here $A$ is assumed to be the generator of an integrated semigroup $\{W(t) : t \geq 0\}$ in $X$ and $B$ a possibly nonlinear operator from a subset $D$ of $Y = \overline{D(A)}$ into $X$. It is assumed that $B$ is continuous on bounded sets with respect to a lower semicontinuous functional $\varphi$ on $X$ such that $D \subset D(\varphi) = \{v \in X; \varphi(v) < \infty\}$. The functional $\varphi$ is also employed to restrict the growth of mild solutions to (SP). The objective of this paper is to discuss generation and characterization of a nonlinear semigroup on $D$ which provides mild solutions to (SP) in the case that the nonlinear operator $B$ is not necessarily quasidissipative. The generation theorem is established

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