Nihonkai Math. J.
Vol.13(2002), 179-182

Growth of transcendental entire solution of some q-difference equation

Ilham Eli, Kayoko Gotoh and Niro Yanagihara

Abstract

We consider a linear q-difference equation $qzf(qz)+(1-Az)f(z)=1$, with $q=e^{2\pi i\beta}, \beta\in(0,1)\setminus\mathbb{Q}$ and $A=e^{2\pi i\alpha}, \alpha\in(0,1)$. The equation is known to admit a transcendental entire solution $f(z)$ for suitably chosen β and α. We will show here that $f(z)$ is of positive order for some β, contrary to q-difference equations with $|q|\neq 0,1$.

Keywords and phrases: linear q-difference equation, growth of entire function.

AMS Subject Classification: 39A13, 30D20

1 Introduction

We consider here a q-difference equation

(1.1) $$b_p(z)f(q^pz)+\cdots+b_0(z)f(z)=\mathfrak{b}(z), \quad b_j(z),\ \mathfrak{b}(z)\in\mathbb{C}[z],$$

with $b_j(z)=\sum_{k=0}^{B_j} b_k^{(j)}z^k\ \ (b_{B_j}^{(j)}\neq 0), 0\leq j\leq p$.

When $|q|\neq 0,1$, a transcendental entire solution $f(z)$ of (1.1) are of order 0. In fact, when $0<|q|<1$, it satisfies

$$\log M(r,f)=\frac{\sigma}{-2\log|q|}(\log r)^2(1+o(1)), \quad r\to\infty,$$

in which σ is a slope of the Newton diagram for (1.1) [1].

When $|q|=1$, that is $q=e^{2\pi i\lambda}$, there is no such regularity. For example, when $q=-1, \lambda=1/2$, the equation $f(-z)-f(z)=0$ has solutions of behaviors of several type. We ask here what can be said for the case that

(1.2) $$q=e^{2\pi i\beta}, \quad \beta\in(0,1)\setminus\mathbb{Q}.$$

Driver et al. [3] showed that there exist (q,A), with q in (1.2) and $A, |A|=1$, such that the equation

(1.3) $$qzf(qz)+(1-Az)f(z)=1$$

has a transcendental entire solution. We will show here the following theorem, contrary to the case $|q|\neq 0,1$:

Theorem 1.1 *The solution $f(z)$ of (1.3) is of positive order, supposed β in (1.2) is suitably chosen, as shown at the end of the proof.*