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THE CONDITIONS THAT THE TOEPLITZ OPERATOR IS NORMAL OR ANALYTIC

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Abstract. P. R. Halmos [6; Problem 5] asked whether every subnormal Toeplitz operators on H^2 was either analytic or normal. A negative example was given by C. C. Cowen and J. J. Long [5; Theorem]. In this paper, we shall give the conditions that the Toeplitz operator T_φ is normal or analytic and show, as their applications, the following results: (1) If T_φ is hyponormal with $\mathcal{N}_{T_\varphi{}^*T_\varphi - T_\varphi T_\varphi{}^*} = \{f \in H^2 \; : \; (T_\varphi{}^*T_\varphi - T_\varphi T_\varphi{}^*)f = o\}$ as its invariant subspace and if $\mathcal{N}_{H_\varphi} \cup \mathcal{N}_{H_{\bar\varphi}} \neq \{o\}$, then T_φ is normal or analytic ([1; Theorem]) and (2) Every quasi-normal Toeplitz operator is only normal or a scalar multiple of an isometry ([2; Theorem]).

1. Preliminaries. A bounded measurable function $\varphi \in L^\infty$ on the circle induces the multiplication operator on L^2 called the **Laurent operator** L_φ given by $L_\varphi f = \varphi f$ for $f \in L^2$. And the Laurent operator induces in a natural way twin operators on H^2 called **Toeplitz operator** T_φ given by $T_\varphi f = PL_\varphi f$ for $f \in H^2$, where P is the orthogonal projection from L^2 onto H^2 and **Hankel operator** H_φ given by $H_\varphi f = J(I-P)L_\varphi f$ for $f \in H^2$, where J is the unitary operator on L^2 defined by $J(z^{-n}) = z^{n-1}$, $n = 0, \pm1, \pm2, \cdots$. The following results are well known.

Proposition 1. ([3; Theorem IV]) If $\mathcal{M}$ is a non-zero invariant subspace of T_z, then there exists an isometric Toeplitz operator T_g uniquely, up to a unimodular constant, such that $\mathcal{M} = T_g H^2$.

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