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Geometry Of Geodesics For Convex Billiards And Circular Billiards

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Dedicated to Professor K. Shiohama on his sixtieth birthday

Abstract

In the present paper circles and ellipses will be characterized by some properties of billiard ball trajectories. Those properties will be discussed in connection with the characterization of flat metrics on tori by some families of geodesics and tori of revolution. The main method is the geometry of geodesics due to H. Busemann which was reconstructed in the configuration space by V. Bangert. In particular, the theory of parallels plays an important role in the present paper.

1 Introduction

Let C be a smooth simple closed and strictly convex curve with length L in the Euclidean plane $\mathbf{E}$ and let $c : \mathbf{R} \longrightarrow \mathbf{E}$ be its representation with arclength, namely $|\dot{c}(t)| = 1$ for any $t \in \mathbf{R}$ where $\mathbf{R}$ is the set of all real numbers. Let $x = (x_j)_{j\in\mathbf{Z}}$ be a sequence of points in C where $\mathbf{Z}$ is the set of all integers. We say that x is a *billiard ball trajectory* if the angle between the tangent vector A to C at x_i and the oriented segment $T(x_{i-1}, x_i)$ from x_{i-1} to x_i is equal to the one between A and $T(x_i, x_{i+1})$ for any $i \in \mathbf{Z}$.

A billiard ball trajectory $x = (x_j)_{j\in\mathbf{Z}}$ in C is represented by a sequence $s = (s_j)_{j\in\mathbf{Z}}$ of real numbers such that $x_j = c(s_j)$ and $s_j < s_{j+1} < s_j + L$ for any $j \in \mathbf{Z}$ and the sequence $s = (s_j)_{j\in\mathbf{Z}}$ will be considered to be a configuration $\{(j, s_j)\}_{j\in\mathbf{Z}}$ in the configuration space $\mathbf{X} = \mathbf{Z} \times \mathbf{R} \subset \mathbf{R}^2$. A configuration $s = (s_j)_{j\in\mathbf{Z}}$ for x is determined uniquely up to the difference pL ($p \in \mathbf{Z}$).