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THE STRUCTURE OF NORM-ACHIEVED TOEPLITZ AND HANKEL OPERATORS

Dedicated to Professor Tsuyosi Andô on his 70th birthday

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Abstract. In this paper, we shall study the structure of norm-achieved Toeplitz and Hankel operators and give their applications for the case where they are paranormal operators. And also we shall prove some property of continuous functions on the unit circle.

A bounded measurable function $\varphi \in L^{\infty}$ on the circle induces the multiplication operator on L^2 called the Laurent operator L_{φ} given by $L_{\varphi}f = \varphi f$ for $f \in L^2$. And the Laurent operator induces in a natural way twin operators on H^2 called Toeplitz operator T_{φ} given by $T_{\varphi}f = PL_{\varphi}f$ for $f \in H^2$, where P is the orthogonal projection from L^2 onto H^2 and Hankel operator H_{φ} given by $H_{\varphi}f = J(I-P)L_{\varphi}f$ for $f \in H^2$, where J is the unitary operator on L^2 defined by $J(z^{-n}) = z^{n-1}$, $n = 0, \pm 1, \pm 2, \cdots$.

The following results are well known.

Proposition 1. For $f \in L^2$, let $f^*(z) = \overline{f(\overline{z})}$ where the bar denotes the complex conjugate. Then $\|f^*\|_2 = \|f\|_2$ and $f^* \in L^2$. Particularly, if $f \in H^2$, then $f^* \in H^2$ also. Moreover, for $\varphi \in L^{\infty}$, $\|\varphi^*\|_{\infty} = \|\varphi\|_{\infty}$ and $\varphi^* \in L^{\infty}$. Particularly, if φ is inner, then φ^* is also inner.

Proposition 2. ([1]) Let $\mathcal{M}$ be an invariant subspace of L_z. Then, in the case where $L_z\mathcal{M} = \mathcal{M}$, there exists a characteristic function χ_E of some measurable subset E of the unit circle such that $\mathcal{M} = L_{\chi_E}L^2$ and, in the case where $L_z\mathcal{M} \subset \mathcal{M}$, there exists a unitary Laurent operator L_g uniquely, except a constant multiple of absolute value one, such that $\mathcal{M} = L_gH^2$.

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