Nihonkai Math. J.
Vol.12(2001), 135-143

STRUCTURE OF GROUP C^*-ALGEBRAS OF SEMI-DIRECT PRODUCTS OF $\mathbb{C}^n$ BY $\mathbb{Z}$

TAKAHIRO SUDO

ABSTRACT. We consider the structure of group C^*-algebras of semi-direct products of $\mathbb{C}^n$ by $\mathbb{Z}$. As an application we estimate the stable rank and connected stable rank of these C^*-algebras, and treat the case of semi-direct products of $\mathbb{R}^n$ by $\mathbb{Z}$ similarly.

§0. INTRODUCTION

Group C^*-algebras have played important roles in the progress of the theory of C^*-algebras. In particular, their structure for Lie groups has been investigated (cf.[Dx], [Rs], [Gr1,2], [Pg], [Wg], etc). On the other hand, the stable rank for C^*-algebras was introduced by M.A. Rieffel [Rf1] as a noncommutative analogue of the covering dimension for topological spaces, and he raised an interesting problem such as describing the stable rank of group C^*-algebras of Lie groups in terms of groups. On this problem some partial answers were obtained by [Sh],[ST1,2] and [Sd1-4]. In particular, in [Sd4] the author investigated the structure of group C^*-algebras of Lie semi-direct products of $\mathbb{C}^n$ by $\mathbb{R}$, and estimated their stable rank and connected stable rank.

In this paper we obtain finite composition series of group C^*-algebras of the semi-direct products of $\mathbb{C}^n$ by $\mathbb{Z}$, by analyzing their subquotients explicitly using some methods of [Sd4] similarly. Using this result we give the rank estimations of these group C^*-algebras, and especially that of semi-direct products of $\mathbb{R}^n$ by $\mathbb{Z}$. These are disconnected solvable (Lie) groups, and contain the discrete Mautner group studied by L. Baggett [Bg] to construct some unitary representations of the Mautner group through Mackey machine. We emphasize that this paper will be the first step to explore the algebraic structure of C^*-algebras of general disconnected solvable Lie groups.

We now prepare some notations. Let $C^*(G)$ be the (full) group C^*-algebra of a locally compact group G (cf.[Dx, Part II],[Pd, Chapter 7]). We denote by $\hat{G}_1$ the space of all 1-dimensional representations of G. Let $C_0(X)$ be the C^*-algebra of all complex valued continuous functions on a locally compact Hausdorff space X vanishing at infinity. When X is compact, we set $C_0(X) = C(X)$. Let $\mathbb{K}$ be the C^*-algebra of all compact operators on a countably infinite dimensional Hilbert space. For a C^*-algebra $\mathfrak{A}$, we denote by $\mathrm{sr}(\mathfrak{A})$, $\mathrm{csr}(\mathfrak{A})$ its stable rank, connected stable rank respectively ([Rf1]).

2000 Mathematics Subject Classification: Primary 46L05, 46L80, 22D25, 19K56.
Key words and phrases: Group C^*-algebras, Semi-direct products, Lie groups, Stable rank
Research partially supported by Japan Society for the Promotion of Science