Nihonkai Math. J.
Vol. 12(2001), 71-73

A NOTE ON UNIQUENESS IN AN INVERSE PROBLEM FOR A SEMILINEAR PARABOLIC EQUATION

SHIN-ICHI NAKAMURA

ABSTRACT. Consider the mixed problem for a semilinear parabolic equation $u_t - \Delta u + a(u) = 0$. Isakov proved the uniqueness result of the function a by prescribing any initial and lateral Dirichlet data and measuring lateral Neumann data and final data under the condition $a(0) = 0$. In this note we shall study the case $a(0) \neq 0$.

1. Introduction. Let Ω be a bounded domain in $\mathbb{R}^n$ $(n \geq 2)$ with a C^2-boundary $\partial\Omega$ and set $Q_T \equiv \Omega \times (0,T)$ in $\mathbb{R}^{n+1}$. Let H be the subspace of function g on $\partial Q_T \backslash \{t = T\}$ which belongs to $C^{2,1}(\partial\Omega \times [0,T]) \cap C^1(\bar{\Omega} \times \{0\})$ and which have $C^{\lambda,\lambda/2}(\bar{Q}_T)$ extensions. We now consider the mixed problem:

$$u_t - \Delta u + a(u) = 0 \quad \text{in } Q_T, \tag{1.1}$$

$$u = g \in H \quad \text{on } \partial Q_T \backslash \{t = T\}, \tag{1.2}$$

where $a(s) \in C^2(\mathbb{R})$ satisfies the conditions:

$$a(s) \text{ and } a_{ss}(s) \text{ are bounded on } \mathbb{R}, \tag{1.3a}$$

$$0 \leq a_s \leq M, \tag{1.3b}$$

where M is a positive constant.

Under the condition (1.3b), there is a unique solution $u \in H^{2,1}(Q_T) \cap C(\bar{Q}_T)$ to the problem (1.1)-(1.2)(Theorem 6.1 in [3, p. 452] and [2]). (The norms and the properties of the function spaces can be found in [2] or [3].) So we may define

$$h = u \quad \text{on } \Omega \times \{T\},\ h = \partial_\nu u \quad \text{on } \partial\Omega \times (0,T),$$

here ν denotes the unit exterior normal to $\partial\Omega$. We are interested in uniqueness results of the function a from the map:

$$\Lambda(a) : g \longmapsto h.$$

Let $\Lambda_j = \Lambda(a^j)$ $(j = 1,2)$. The following theorem can be derived from Theorem 1 in [2].