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On Ricci curvature of CR-submanifolds with rank one totally real distribution

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Abstract

In a recent paper, Bang-yen Chen obtained sharp inequalities between the maximum Ricci curvature and the squared mean curvature for arbitrary submanifolds in real space forms and totally real submanifolds in complex space forms ([6, 7]). In this paper we give sharp inequalities between the maximum Ricci curvature and the squared mean curvature for arbitrary submanifolds in complex space form. Moreover we investigate CR-submanifolds in complex space forms and in the nearly Kaehler six-sphere which realize the equality case of the inequalities mentioned above.

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1 Introduction

Let M^n be an n-dimensional submanifold of an m-dimensional manifold $\tilde{M}^m$. Denote by h the second fundamental form of M^n in $\tilde{M}^m$. Then the mean curvature vector $\vec{H}$ of the immersion is given by $\vec{H}=\frac{1}{n}\operatorname{trace} h$. A submanifold is said to be minimal if its mean curvature vector vanishes identically. Denote by D the linear connection induced on the normal bundle $T^{\perp}M^n$ of M^n in $\tilde{M}^m$, by R and $\tilde{R}$ the Riemann curvature tensors of M and of $\tilde{M}^m$ respectively, and by R^D the curvature tensor of the normal connection D. Then the equation of Gauss and Ricci are given respectively by

$$R(X,Y)Z \;=\; \left\langle A_{h(Y,Z)}X, W\right\rangle - \left\langle A_{h(X,Z)}Y, W\right\rangle + \tilde{R}(X,Y)Z \tag{1.1}$$

$$R^D(X,Y;\xi,\eta) = \tilde{R}(X,Y;\xi,\eta) + \langle [A_\xi, A_\eta](X), Y\rangle \tag{1.2}$$

for vectors X,Y,Z,W tangent to M and ξ,η normal to M, where A is the shape operator. For the second fundamental form h, we define the covariant derivative $\bar{\nabla}h$ of h with respect to the connection on $TM\oplus T^{\perp}M$ by

$$(\bar{\nabla}_X h)(Y,Z) = D_X(h(Y,Z)) - h(\nabla_X Y, Z) - h(Y,\nabla_X Z). \tag{1.3}$$

The equation of Codazzi is given by

$$(\tilde{R}(X,Y)Z)^{\perp} = (\bar{\nabla}_X h)(Y,Z) - (\bar{\nabla}_Y h)(X,Z). \tag{1.4}$$

The Ricci tensor S and the scalar curvature τ at a point $p\in M^n$ are given respectively by $S(X,Y)=\sum_{i=1}^n\langle R(e_i,X)Y,e_i\rangle$ and $\tau=\sum_{i=1}^n S(e_i,e_i)$, where $\{e_1,\dots,e_n\}$ is an orthonormal basis of the tangent space T_pM^n.