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ON SPACE-LIKE KÄHLER SUBMANIFOLDS WITH $RS = 0$ IN AN INDEFINITE COMPLEX SPACE FORM

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ABSTRACT. In this paper we give some characterizations of Kähler manifolds and complete space-like complex submanifolds with the Ryan condition $RS = 0$ in an indefinite complex hyperbolic space.

§1. Introduction

As well known, Ryan [11] investigated complex hypersurfaces in a complex space form satisfying the condition

$$R(X,Y)S = 0 \tag{1.1}$$

for any vector fields X and Y tangent to the hypersurface M, where R denote the Riemannian curvature tensor, S is the Ricci tensor on M and $R(X,Y)$ operates on the tensor algebra as a derivation. The condition (1.1) is called the Ryan one. Relative to the Ryan condition, Ryan [11] proved that these hypersurfaces are Einstein manifolds if the holomorphic sectional curvature of the ambient space does not vanish, which was generalized from two distinct directions. One of them is due to Takahashi [12], who verified that such hypersurfaces become cylindrical if the ambient space is complex Euclidean. Another extension is treated by Kon [7] in the case of complex submanifolds in a complex space form of constant negative holomorphic sectional curvature. On the other hand, independently of Kon's work, Aiyama, Kwon and Nakagawa [1] researched about properties on space-like complex submanifolds satisfying the Ryan condition in an indefinite complex space form. In the case of complex submanifolds in a complex space form of constant positive holomorphic sectional curvature, these submanifolds were determined by Nakagawa and Takagi [8].

On the other hand, Ki and Suh [4] observed the Ryan condition from the different point of view and obtained a nice theorem about Kähler manifolds whose totally real bisectional curvature is bounded from below by a positive constant. Thus it seems to us to be interesting to investigate the space-like Kähler submanifolds satisfying the Ryan condition of an indefinite complex space form.

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