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CHARACTERIZATION OF POSINORMAL OPERATORS

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Abstract

In this paper, we give a characterization of posinormal operators. And we introduce a new class of operators and show M-paranormality of such operators.

1. Introduction.

In [1], H.C. Rhaly Jr. introduced and studied posinormal operators. He showed a characterization of posinormality and spectral properties of posinormal operators. Moreover, he gave many fruitful examples of posinormal operators for the Casáro operator. In this paper, first we give another characterization of posinormality. Next we introduce p-posinormal operators and give a characterization of it. Finally, we show that p-posinormal operators are M-paranormal.

Let $\mathcal{H}$ be a complex separable Hilbert space and $\mathrm{B}(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. In what follows, an operator means a bounded linear operator on $\mathcal{H}$. An operator T is said to be a posinormal operator if there exists a positive operator $P \in B(\mathcal{H})$ such that $TT^* = T^*PT$. Here, an operator P just mentioned above is called an interrupter of T. The set of all posinormal operators in $\mathrm{B}(\mathcal{H})$ is denoted by $P(\mathcal{H})$ (see [1]). Let p be $0 < p \leq 1$. An operator T is said to be a p-hyponormal operator if $(T^*T)^p \geq (TT^*)^p$. An operator $T \in B(\mathcal{H})$ is said to be p-posinormal if $(TT^*)^p \leq \lambda^2(T^*T)^p$ for some positive number λ. We denote the set of all p-posinormal operators by p-$P(\mathcal{H})$. By Rhaly's characterization of posinormality, we can see that 1-posinormal operators are posinormal (cf. Theorem B). According to [3], an operator $T \in B(\mathcal{H})$ is said to be M-paranormal if there exists $\lambda > 0$ such that $\|Tx\|^2 \leq \lambda\|T^2x\|$ for all $x \in \mathcal{H}$ with $\|x\| = 1$. Let call P an interrupter of T with degree p if $|T^*|^{2p} = |T|^pP|T|^p$.

2. Result.

First, we need the following theorems.