

LAPLACE OPERATOR ON COMPACT GLUED MANIFOLDS

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1. INTRODUCTION

Various studies are developed on the property of the Laplace operator on Riemannian manifolds. In this paper we introduce the concept of *glued manifold* (§2) as an extension of smooth Riemannian manifolds and develop fundamental properties of the Laplace operator on it. Roughly speaking, a glued manifold is a manifold which consists of several pieces of Riemannian manifolds with boundary glued together by isometric mappings between boundaries. For instance, this kind of manifold naturally appears as surface of an object placed in Euclidean space E^3 (See FIGURE 1). On a glued manifold M the Riemannian metric g is thought to be continuous on M but it is thought not to be differentiable on each point of boundaries on which two pieces of Riemannian manifolds are glued together. A glued manifold is an elementary example of a Lipschitz manifold [CP89] [Tel83].

We find that the Laplace operator on functions can be defined on a compact glued manifold (§3). And we find that L^2 -space of functions has a orthogonal decomposition into eigen spaces of the Laplace operator just same as on compact smooth Riemannian manifolds (Theorem 1). These results are shown in [CP89] for general compact Lipschitz manifolds, however,

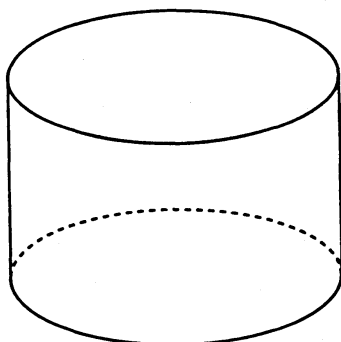


FIGURE 1

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