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SUBDIAGONAL ALGEBRAS IN NON-σ-FINITE VON NEUMANN ALGEBRAS

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ABSTRACT. Let $\mathfrak{A}$ be a subdiagonal algebra of a von Neumann algebra $\mathcal{M}$, which is not σ-finite, with respect to a faithful normal expectation Φ. In this note we generalize some results of subdiagonal algebras in the σ-finite case to the non-σ-finite case. We prove that there is a unique maximal subdiagonal algebra $\mathfrak{A}_m$ with respect to Φ containing $\mathfrak{A}$. We show that if $\mathfrak{A}$ is maximal subdiagonal and φ is a faithful normal semi-finite weight on $\mathcal{M}$ such that $\varphi \circ \Phi = \varphi$, then $\mathfrak{A}$ is σ_t^{φ}-invariant $(\forall t \in \mathbb{R})$, where $\{\sigma_t^{\varphi}\}_{t\in\mathbb{R}}$ is the modular automorphism group associated with φ. As an application, we also give several characterizations of $\mathfrak{A}_m$.

1. INTRODUCTION

In [1], Arveson introduced the notion of subdiagonal algebras in a von Neumann algebra on a Hilbert space to study the analyticity in operator algebras. At first, we start by given the definition of subdiagonal algebras. Let $\mathcal{M}$ be a von Neumann algebra on a complex Hilbert space $\mathcal{H}$, and let Φ be a faithful normal positive idempotent linear map from $\mathcal{M}$ onto a von Neumann subalgebra $\mathfrak{D}$ of $\mathcal{M}$. A subalgebra $\mathfrak{A}$ of $\mathcal{M}$, containing $\mathfrak{D}$, is called a subdiagonal algebra in $\mathcal{M}$ with respect to Φ if

(i) $\mathfrak{A} \cap \mathfrak{A}^* = \mathfrak{D}$,

(ii) Φ is multiplicative on $\mathfrak{A}$, and

(iii) $\mathfrak{A} + \mathfrak{A}^*$ is σ-weakly dense in $\mathcal{M}$.

The algebra $\mathfrak{D}$ is called the diagonal of $\mathfrak{A}$. Although subdiagonal algebras are not assumed to be σ-weakly closed in [1], the σ-weak closure of a subdiagonal algebra is again a subdiagonal algebra([1, Remark 2.1.2]). Thus we assume that our subdiagonal algebras are always σ-weakly closed. We say that $\mathfrak{A}$ is a maximal subdiagonal algebra in $\mathcal{M}$ with respect to Φ in case $\mathfrak{A}$ is not properly contained in any other subalgebra of $\mathcal{M}$ which is subdiagonal with respect to Φ. Put $\mathfrak{A}_0 = \{X \in \mathfrak{A} : \Phi(X) = 0\}$, and let $\mathfrak{A}_m$ be the set of all $A \in \mathcal{M}$ such that $\Phi(\mathfrak{A}A\mathfrak{A}_0) = \Phi(\mathfrak{A}_0 A\mathfrak{A}) = 0$. Arveson has proved that $\mathfrak{A}_m$ is the unique maximal subdiagonal algebra with respect to Φ

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