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ON PSEUDO-UMBILICAL SURFACES WITH NONZERO PARALLEL MEAN CURVATURE VECTOR IN $\mathbb{C}P^3(\tilde{c})$ II

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Abstract. In this paper, we classify pseudo-umbilical surfaces in a complex 3-dimensional complex projective space under some additional condition.

1. INTRODUCTION

Let $\mathbb{C}P^m(\tilde{c})$ be a complex m-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature $\tilde{c}$. The class of totally umbilical submanifolds in $\mathbb{C}P^m(\tilde{c})$ was completely classified by Chen and Ogiue [1]. However, it is well known that the class of pseudo-umbilical submanifolds in $\mathbb{C}P^m(\tilde{c})$ is too wide to classify. Thus, it is reasonable to study pseudo-umbilical submanifolds in $\mathbb{C}P^m(\tilde{c})$ under some additional condition.

Recently, the author [5] proved the following Theorem.

Theorem A. *Let M be an $n(\geq 2)$-dimensional pseudo-umbilical submanifold with nonzero parallel mean curvature vector in $\mathbb{C}P^m(\tilde{c})$. If $2m - n \geq 2$, then $m > n$ and M^n is immersed in $\mathbb{C}P^m(\tilde{c})$ as a totally real submanifold.*

Immediately, we see that $\mathbb{C}P^2(\tilde{c})$ admits no pseudo-umbilical surfaces with nonzero parallel mean curvature vector. The aim of this paper is to classify pseudo-umbilical surfaces with nonzero parallel mean curvature vector in $\mathbb{C}P^3(\tilde{c})$. Now we get the following Theorem.

Theorem 1.1. *Let $M(K)$ be a complete pseudo-umbilical surface of constant Gauss curvature K with nonzero parallel mean curvature vector ζ in $\mathbb{C}P^3(\tilde{c})$. Then $M(K)$ is one of the following:*

(1) *$M(K)$ is an extrinsic hypersphere in a 3-dimensional real projective space $\mathbb{R}P^3(\tilde{c}/4)$ of $\mathbb{C}P^3(\tilde{c})$.*
(2) *$M(K)$ is a constant isotropic totally real surface in $\mathbb{C}P^3(\tilde{c})$ and the covariant derivative $\bar{\nabla}\sigma$ of the second fundamental form σ is proportional to $J\zeta$.*

Remark 1.1. By Proposition 4.1, we can descrive the covariant derivative $\bar{\nabla}\sigma$ of the second fundamental form σ of the surface (2) in Theorem 1.1 explicitly.

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