Nihonkai Math. J.
Vol.10 (1999), 117–130

Leaf space of a certain Hopf r-foliation

Shinsuke YOROZU

Abstract

The Hopf r-foliation $\mathcal{F}^r$ on S^3 is a generalization of the classical Hopf fibration of S^3. When r is an integer and is greater than 1, we describe the leaf space $S^3/\mathcal{F}^r$ of the Hopf r-foliation as a surface of revolution $(\mathrm{S}_r, ds^2_{\mathrm{S}_r})$ in $(R^3, ds^2_{R^3})$. Then the natural projection $\tilde{p} : (S^3, ds^2_{S^3}) \longrightarrow (\mathrm{S}_r, ds^2_{\mathrm{S}_r})$ becomes a C^∞ Riemannian V-submersion.

1 Introduction

For a given positive number r, we consider a foliation $\mathcal{F}^r$ defined on the unit 3-sphere S^3 whose leaves are given by the flow

$$\gamma_t^r(z,w) = (e^{irt}z, e^{it}w), \qquad (z,w) \in S^3, \quad t \in R$$

on $S^3 \subset \mathbf{C}^2$([1,10]). We call $\mathcal{F}^r$ the Hopf r-foliation on S^3 ([10]). It should be remark that the Hopf 1-foliation $\mathcal{F}^1$ on S^3 is the one given by the classical Hopf fiberation of S^3. If r is a rational number, then each leaf of $\mathcal{F}^r$ is closed and the canonical metric $ds^2_{S^3}$ on S^3 is a bundle-like metric with respect to $\mathcal{F}^r$. Thus the leaf space $S^3/\mathcal{F}^r$ becomes a C^∞ Riemannian V-manifold([7,8]). See Satake[9] for the notion of V-manifolds. When r is an integer and is greater than 1, we can realize the leaf space $S^3/\mathcal{F}^r$ as a surface of revolution $(\mathrm{S}_r, ds^2_{\mathrm{S}_r})$ in a Euclidean 3-space R^3,where $ds^2_{\mathrm{S}_r}$ is the metric induced from the canonical metric $ds^2_{R^3}$ on R^3. A parametrization of the surface S_r is given explicitly in section 3. Consequently, the natural projection $p : S^3 \longrightarrow S^3/\mathcal{F}^r$ induces a mapping $\tilde{p} : S^3 \longrightarrow \mathrm{S}_r$. Then our main theorem in this paper is

Theorem. *Let r be an integer and suppose $r > 1$. Let $\mathcal{F}^r$ be the Hopf r-foliation on S^3. Then the leaf space $S^3/\mathcal{F}^r$ is homeomorphic to the surface of revolution S_r in R^3, and the mapping $\tilde{p} : (S^3, ds^2_{S^3}) \longrightarrow (\mathrm{S}_r, ds^2_{\mathrm{S}_r})$ is a C^∞ Riemannian V-submersion.*

When r is a positive rational number and is not an integer, we can also construct a surface of revolution $(\hat{\mathrm{S}}_r, ds^2_{\hat{\mathrm{S}}_r})$ and obtain a C^∞ V-submersion $\hat{p} : S^3 \longrightarrow \hat{\mathrm{S}}_r$. However, $\hat{p} : (S^3, ds^2_{S^3}) \longrightarrow (\hat{\mathrm{S}}_r, ds^2_{\hat{\mathrm{S}}_r})$ is not a C^∞ Riemannian V-submersion (Remark in section 4).