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Cartan hypersurfaces and reflections

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Abstract

One gives a characterization of the Cartan hypersurfaces in spheres by means of volume-preserving local reflections.

1. Introduction and statement of the results

In this short note we will treat some geometrical properties of a special class of minimal hypersurfaces M embedded in a sphere $S^{n+1}(c)$ of curvature c. We always suppose M to be connected and compact.

We start with the definition of this class.

Definition. A *Cartan hypersurface* in a sphere $S^{n+1}(c)$ is a compact hypersurface with principal curvatures $-(3c)^{1/2}$, 0, $(3c)^{1/2}$ with the same multiplicity.

These hypersurfaces were discovered by E.Cartan in his work about isoparametric hypersurfaces in real space forms [2], [3]. First, he discovered the socalled classical Cartan hypersurface in $S^4(1)$. It is the only complete hypersurface, up to congruence, with three distinct constant principal curvatures. Further, it is an "algebraic" manifold defined by a polynomial of order three. It is minimally embedded and moreover, it is a homogeneous space $SO(3)/\mathbf{Z}_2 \times \mathbf{Z}_2$ which may be viewed as a tube of radius $\pi/2$ about a Veronese surface. (See also [6] for a description.) Next, E.Cartan also proved that these hypersurfaces only exist when $n = 3, 6, 12, 24$ and that the compact ones are always homogeneous.

Many authors studied *isoparametric hypersurfaces*, i.e. hypersurfaces with constant principal curvatures, in real space forms. Every family of isoparametric hypersurfaces contains a unique minimal one and the Cartan hypersurfaces are the compact ones where there are exactly three distinct principal curvatures. In the reference list we give

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