Nihonkai Math. J.
Vol.1(1990), 193-201

TOTALLY REAL SUBMANIFOLDS OF S^6

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ABSTRACT: We obtain an estimate for the index of a 3-dimensional compact totally real submanifold of the nearly Kaehler six dimensional sphere S^6.

A six dimensional unit sphere S^6 has an almost complex structure J defined by the vector cross product in the space of purely imaginary cayley numbers. This almost complex structure is not integrable and satisfies $(\bar{\nabla}_{\bar{X}} J)(\bar{X}) = 0$ for any vector field $\bar{X}$ on S^6, where $\bar{\nabla}$ is the Riemannian connection on S^6 (and hence S^6 is a nearly Kaehler manifold) (cf. [2]). It is also known that there does not exist a 4-dimensional almost complex submanifold of S^6 (cf. [2]). However there are 3-dimensional totally real submanifolds of S^6 (cf. [1]). 3-dimensional totally real submanifolds of S^6 are minimal and orientable ([1]).

Let M be a 3-dimensional totally real submanifold of S^6 with the tangent bundle TM and the normal bundle ν. We denote by the same letter g the Riemannian metric on S^6 as well as that induced on M. The Riemannian connection $\bar{\nabla}$ induces the Riemannian connection ∇ on M and the connection $\nabla^{\perp}$ in the normal bundle ν and we have the following Gauss and Weingarten formulae

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This work is supported by the grant No.(Math/1409/04) of the Research Center, College of Science, King Saud University, Riyadh.