Shifts with two generators on the hyperfinite $\mathbb{II}_1$-factor

Keiichi Watanabe

1. Introduction

R. T. Powers([6]) introduced a concept of a shift on the hyperfinite $\mathbb{II}_1$-factor $\mathfrak{R}$, which is an identity preserving *-endmorphism σ such that $\cap_{k=1}^{\infty}\sigma^k(\mathfrak{R}) = \mathbb{C}1$. He defined the index of σ as the Jones index $[\mathfrak{R}:\sigma(\mathfrak{R})]$. He discussed on conjugacy or on outer conjugacy of binary shifts which is a class of shifts of index two on $\mathfrak{R}$. A shift σ on $\mathfrak{R}$ is said to be a binary shift if there is a unitary element $u \in \mathfrak{R}$ with $u^2 = 1$ which satisfies $\mathfrak{R} = \{\sigma^k(u) ; k \geq 0\}''$ and $u\sigma^k(u) = \pm\sigma^k(u)u$ for $k \in \mathbb{N}$. There are uncountably many non conjugate, at least countably many non outer conjugate binary shifts on $\mathfrak{R}$. Enomoto, Choda and Watatani considered a general shift σ on a group von Neumann algebra $R_m(G)$ on a group G twisted by a multiplier m such that the shift σ is induced from a shift on G, and they generalized results of Powers' binary shifts. Bures and Yin also independently studied the shifts as mentioned above.

In this paper we consider a class of shifts which have two generators in a sense. At first, we shall show that a shift with two