Nihonkai Math. J.
Vol.1(1990),43-53

A Discrete Analog of Laplace's Differential Equations

SHIN TONG TU

1. Intoduction

A differential equation of the form

$$(a_2t+b_2)y''(t)+(a_1t+b_1)y'(t)+(a_0t+b_0)y(t)=0 \tag{1.1}$$

was studied by Laplace in his treatise "Theorie analytique des probabilities"(cf. Yosida [4,p.53]), so that (1.1) may be called the Laplace differential equations. In a recent monograph [4],Yosida gives a new treatment of (1.1) by his operational calculus method. In the present paper, we shall study a discrete analog of the Laplace differential equation (1.1), namely, the monodiffric difference equation of the form

$$K(z)\circ\frac{d^2f}{dz^2}+(a_1H(z)+b_1)\circ\frac{df}{dz}+(a_0G(z)+b_0)\circ f(z)=0. \tag{1.2}$$

We shall use the formal series method to find the general solution of (1.2). Our main result is Theorem 2. And Theorem 2 can be applied to solve Bessel, Laguerre and Gauss monodffric difference equations which will be introduced in Section 6.

2. Definition and Notation

For the sake of convenience,we give some definitions and notations which are mentioned in [3]. Let $\mathbf{C}$ be the complex plane,

$$D=\{z\in\mathbf{C}\mid z=x+iy,\, x \text{ and } y \text{ are integers }\}.$$

DEFINITION 1. The function $f:D\to\mathbf{C}$ is said to be monodiffric at z if

$$(i-1)f(z)+f(z+i)-if(z+1)=0. \tag{2.1}$$

The function f is said to be monodiffric in D if it is monodiffric at any point in D.

DEFINITION 2. The monodiffric derivative f' of f is defined by

$$f'(z)=\frac{1}{2}[(i-1)f(z)+f(z+1)-if(z+i)]. \tag{2.2}$$

We also use the symbol $\frac{df}{dz}$ to represent f'.

DEFINITION 3. Suppose that f and g are complex-valued functions defined on D. Let $z\in D$ and $h\in\{1,i,-1,-i\}$. The line integrals from z to $z+h$ are defined respectively by