On 4-dimensional connected Einstein spaces satisfying the condition R(X, Y) · R=0

By

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1. Introduction

Let M be a 4-dimensional connected Einstein space with the Ricci tensor $S=\lambda g$, where g is the Riemannian metric of M and λ is a constant.

In this paper, we show the following theorem

THEOREM 1. 1 *Let M be a 4-dimensional connected Einstein space. Assume that*

(1. 1) $\qquad R(X, Y)\cdot R=0 \qquad$ *for all tangent vectors X and Y.*

Then, $\nabla R=0$, *that is, M is locally symmetric.*

Now, we can see that there is an orthonormal basis $\{e_1, e_2, e_3, e_4\}$ at each tangent space of M such that

(1. 2)
$$\begin{array}{lll} R_{1212}=a, & R_{1313}=b, & R_{1414}=c, \\ R_{2323}=c, & R_{2424}=b, & R_{3434}=a, \\ R_{1234}=f, & R_{1342}=h, & R_{1423}=-(f+h), \end{array}$$

otherwise zero. Where, $R_{ijkl}=g(R(e_i, e_j)e_k, e_l)$, $1\leqq i, j, k, l,\leqq 4$.
And, as M is an Einstein space with the Ricci curvature λ, the relation

(1. 3) $\qquad a+b+c=-\lambda, \qquad$ holds good.

As the endomorphism $R(X, Y)$ operates on R as a derivation of the tensor algebra at each point of M, (1. 1) implies

(1. 4) $\qquad [R(e_i, e_j), R(e_k, e_l)]=R(R(e_i, e_j)e_k, e_l)+R(e_k, R(e_i, e_j)e_l)$

2. Proof of theorem

First we state a lemma