On complex hypersurfaces of spaces of constant holomorphic sectional curvature satisfying a certain condition on the curvature tensor

By

Kouei SEKIGAWA

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1. Introduction

If a Riemannian manifold M is locally symmetric, then its curvature tensor R satisfies

(*) $$R(X, Y)\cdot R=0 \qquad \text{for all tangent vectors } X \text{ and } Y$$

where the endomorphism $R(X, Y)$ operates on R as a derivation of the tensor algebra at each point of M. Conversely, does this algebraic condition on the curvature tensor field R imply that M is locally symmetric?
We conjecture that the answer is affirmative in the case where M is a complete and irreducible and dim $M\geqq 3$.

The main purpose of the present paper is to consider the complex hypersurfaces in spaces of constant holomorphic sectional curvature satisfying the condition (*) on the curvature tensor.

2. Complex space forms

A Riemannian manifold M with Riemannian metric g is called an Einstein manifold if its Ricci tensor S satisfies $S=\rho g$, where ρ is a constant. We call ρ the Ricci curvature of the Einstein manifold.

Let M be a complex analytic manifold of complex dimension n. By means of charts we may transfer the complex structure of complex n-dimensional Euclidean space C^n to M to obtain an almost complex structure J on M, i.e., a tensor field J on M of type (1.1) such that $J^2=-I$, where I is the tensor field which is the identity transformation on each tangent space of M.

A Riemannian metric g on M is a Hermitian metric if $g(JX, JY)=g(X, Y)$ for any vector fields X and Y on M; M is called a Hermitian manifold. If in addition