Almost contact hypersurfaces in almost Hermitian manifolds

By

Mitsugi NAGAO

(Received June, 1968)

1. Preliminaries

When, in a 2n-dimensional real differentiable manifold M^{2n} with local coordinates $\{x^\lambda\}$, there is given a mixed tensor field $F_\mu{}^\lambda$ satisfying $F_\mu{}^\nu F_\nu{}^\lambda = -\delta_\mu^\lambda$, we say that the manifold admits an almost complex structure $F_\mu{}^\lambda$ and we call such a manifold an almost complex manifold. Throughout the present paper the Greek indices take the values 1, 2, ..., 2n. If an almost complex manifold has a positive definite Riemannian metric tensor $G_{\mu\lambda}$ satisfying $F_\mu{}^\kappa F_\lambda{}^\nu G_{\kappa\nu} = G_{\mu\lambda}$, then the manifold is called an almost Hermitian manifold. In this case it is easily seen that $F_{\mu\lambda} = -F_{\lambda\mu}$, where $F_{\mu\lambda} = F_\mu{}^\nu G_{\nu\lambda}$.

Next, we shall give the definitions of various almost Hermitian manifolds [2]. If, in an almost Hermitian manifold, its structure tensor $F_\mu{}^\lambda$ satisfies

(1.1) $\nabla_\mu F_\lambda{}^\mu = 0$,

(1.2) $\nabla_\beta F_{\alpha\lambda} = -F_\beta{}^\nu F_\alpha{}^\mu \nabla_\nu F_{\mu\lambda}$ (i. e. $\nabla_\nu F_{\mu\lambda}$ is pure in ν and μ),

(1.3) $\nabla_\nu F_{\mu\lambda} + \nabla_\mu F_{\lambda\nu} + \nabla_\lambda F_{\nu\mu} = 0$,

(1.4) $\nabla_\nu F_{\mu\lambda} + \nabla_\mu F_{\nu\lambda} = 0$,

then the manifold is called an almost semi-Kählerian manifold, an *O-manifold, an almost Kählerian manifold (an H-manifold) or an almost Tachibana manifold (a K-manifold) respectively.

If the Nijenhuis tensor $N_{\nu\mu}{}^\lambda$ defined by

$$N_{\nu\mu}{}^\lambda = F_\nu{}^\sigma(\nabla_\sigma F_\mu{}^\lambda - \nabla_\mu F_\sigma{}^\lambda) - F_\mu{}^\sigma(\nabla_\sigma F_\nu{}^\lambda - \nabla_\nu F_\sigma{}^\lambda)$$

vanishes identically, the almost Hermitian manifold, the almost semi-Kählerian manifold and the *O-manifold are called an Hermitian manifold, a semi-Kählerian manifold and a Kählerian manifold respectively [6]. A necessary and sufficient condition for an almost Hermitian manifold to be a Kählerian manifold is given by

(1.5) $\nabla_\nu F_\mu{}^\lambda = 0$.

And it is well-known that in an *O-manifold the two conditions $\nabla_\mu F_\nu{}^\lambda = 0$ and