On complex hypersurfaces of C^{n+1} satisfying a certain condition on the curvature tensor

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1. Introduction

If a Riemannian manifold is locally symmetric, then its curvature tensor R satisfies

$$(*) \qquad R(X, Y)\cdot R=0$$

for all tangent vectors X and Y, where the endomorphism $R(X, Y)$ operates on R as a derivation of tensor algebra at a point of M.

Conversely, does this algebraic condition (*) on the curvature tensor field R imply that M is locally symmetric (i. e. $\nabla R=0$)? In fact, if M is a compact Einstein space, then the statement above is affirmative[1].

K. Nomizu has conjectured that the answer is affirmative in the case where M is irreducible and complete and dim. $M \geqq 3$. And recently he [2] gived an affirmative answer in the case where M is a complete hypersurfac in a Enclidean space.

In this paper, we shall consider a complex hypersurface of C^{n+1} such that its curvature tensor R satisfies (*) and we shall see that the type number at any point of this manifold is 0 or 2. This result will lead directly to the main theorem by virtue of the result by B. Smith [3]. In §2, we shall state some properties of a complex hypersurface of a Kähler manifold and then we shall confine our attention to a complex hypersurface of complex n+1-dimensional Euclidean space C^{n+1} endowed with the usual flat Kähler structure.

§3 will be devoted to the proof of our main theorem.

1) see for example [1]