On hypersurfaces in spaces of constant curvature satisfying a certain condition on the curvature tensor

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1. Introduction

If a Riemannian manifold M is locally symmetric, then its curvature tensor R satifies

(*) $\quad R(X, Y)\cdot R=0 \quad$ for all tangent vectors X and Y,

where the endomorphism $R(X, Y)$ operates on R as a derivation of the tensor algebra at each point of M. Conversely, does this algebraic condition on the curvature tensor field R imply that M is locally symmetric?

We conjecture that the answer is affirmative in the case where M is irreducible and complete and dim $M\geqq 3$.

Recently, K. Nomizu [4], has given an affirmative answer in the case where M is a complete hypersurface in a Euclidean space.

In this paper, let $\tilde{M}$ be a $(n+1)$-dimensional connected pseudo-Riemannian manifold with constant curvature c, and the main purpose is to consider the hypersurfaces of $\tilde{M}$ satisfying the condition (*).

Now, we give a short summary of those parts of the theory of hypersurfaces which are necessary for what follows.

Let M be a real hypersurface immersed in $\tilde{M}$ and g be the induced pseudo-Riemannian metric from the pseudo-Riemannian metric $\tilde{g}$ of $\tilde{M}$. And let H be the second fundamental form with respect to this immersion and A be a field of endomorphism which corresponds to H, that is, $H(X, Y)=g(AX, Y)$, where X and Y are tangent vectors to M.

By definition, the directions of the lines of curvature of M are given by the vectors $\rho_m{}^i$ which satisfy

$$(1.1) \qquad (H_{ij}-\lambda_m g_{ij})\rho_m{}^i=0, \qquad \text{for } m,\ i,\ j=1,2,\dots,n.$$

where H_{ij} and g_{ij} are the components of H and g, respectively, and λ_m is a principal