On extended almost analytic vectors and tensors in almost complex manifolds

By

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1. Extended contravariant almost analytic vectors.

S. Tachibana [9] generalized the notion of contravariant analytic vectors in a Kählerian manifold to an almost Kählerian manifold with structure tensor $\varphi_j{}^i$ and called v^i a contravariant almost analytic vector if it satisfies

$$\underset{v}{£}\varphi_j{}^i \equiv v^r\nabla_r\varphi_j{}^i - \varphi_j{}^r\nabla_r v^i + \varphi_r{}^i\nabla_j v^r = 0 \tag{1.1}$$

where ∇ denotes the operator of covariant derivative with respect to the Riemannian connection. But this formula (1.1) is the so called concomitant and so it is independent of connection. Since, if we consider (1.1) in a Kählerian manifold, it means v^i is an analytic vector, a contravariant almost analytic vector is a generalization of a contravariant analytic vector in a Kählerian manifold. From this point of view, in an almost complex manifold, we shall call v^i an extended contravariant almost analytic vector if it satisfies

$$\underset{v}{£}\varphi_j{}^i + \lambda\varphi_j{}^r N_{rl}{}^i v^l = 0 \tag{1.2}$$

where $N_{rl}{}^i$ is the Nijenhuis tensor and λ is C^∞ scalar function. This vector is also a generalization of a contravariant analytic vector in a Kählerian manifold. In fact, in a Kählerian manifold, since $N_{rl}{}^i = 0$ and $\nabla_j\varphi_i{}^h = 0$, (1.2) shows that v^i is an analytic vector [15].

Particularly, when $\lambda = -\frac{1}{2}$, this definition coincides with Sato's definition obtained from the standpoint of cross-section of a tangent bundle [4].

2. Properties of extended contravariant almost analytic vectors in K-space

By K-space (Tachibana space) we mean a Hermitian manifold M such that

$$\nabla_j\varphi_{ih} + \nabla_i\varphi_{jh} = 0. \tag{2.1}$$