Remarks on certain 14-manifolds

By

Tsuyoshi Watabe

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1. Introduction

The object of this note is to give the classification up to diffeomorphism of closed, 5-connected 14-manifolds. All of our results are valid only for manifolds with torsion free homology which are boundaries of certain 15-manifolds. The proofs of our rsults are straightforward applications of the results of [6] and [8].

Throughout this note, we are only concerned with 14-manifolds M which satisfy the hypothesis;

(H) M is closed, 5-connected and the homology of M is torsion free.

By an (H)-manifold, we shall mean a 14-manifold satifying the hypothesis (H).

2. Splitting theorem

Theorem 1. *Let M be an (H)-manifold. Then we can write M as a connected sum*

$$M=M_1\#(S^7\times S^7)\#\cdots\cdots\#(S^7\times S^7),$$

where M_1 is an (H)-manifold with $H_7(M_1)=0$.

Since the proof of this is analogous to that of theorem 1 in [8], we shall give an outline of the proof.

It is known that $H_7(M)$ admits a symplectic basis $\{e_i,\ e_i'\}$ $(1\leqq i\leqq k)$ so that

$$e_i\cap e_j = e_i'\cap e_j' = 0$$

and

$$e_i\cap e_j' = \delta_{ij}.$$

Since M is 5-connected, the Hurwicz homomorphism $H:\pi_7(M)\longrightarrow H_7(M)$ is an epimorphism. Thus we have mappings $\overline{f_i}$ and $\overline{f_i'}$ of S^7 in M which represent e_i and e_i, respectively. By a theorem of Haefliger [2], a general position argument and the method of Whithey, we may assume that for each i, $\overline{f_i}$ and $\overline{f_i'}$ are embeddings, the image spheres meet each other transversely in a finite set of points, none of which lies on more than two of the spheres, and for each i, $\overline{f_i}$ and $\overline{f_i'}$ intersect in a single point and others do not intersect. Let p_0 be the base point of S^7. We