Bundles contained in fibre bundles

By

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In the present paper we define microbundles and vector bundles admitted by cross-sections of fibre bundles, and study relations between properties of fibre bundles, cross-sections and their admitted bundles.

Milnor has defined piecewise linear microbundles and obtained theorems analogous to those about vector bundles, and applied them to the smoothing problem of piecewise linear manifolds in [3]. As he noted, the definitions and many of the theorems make sense in the category of topological spaces and maps. Letely their detailes are stated in [6].

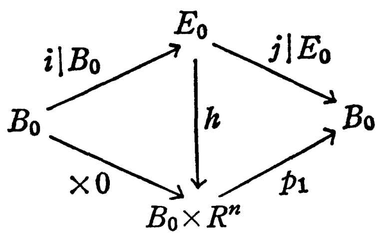
We recall in §1 the definitions required in the later, study about microbunles admitted by cross-sections in §2, study about vector bundles admitted by cross-sections in §3.

1. Microbundles

DEFINITION 1.1. A *microbundle* $\mathfrak{x}$ of dimension n is a diagram

$$B \xrightarrow{\ i\ } E \xrightarrow{\ j\ } B$$

where B, E are topological spaces and i, j are continuous maps; such that the following local triviality condition is satisfied. For each $b \in B$, there should exist neighborhoods B_0 of b, E_0 of $i(b)$ and a homeomorphism $h: E_0 \longrightarrow B_0 \times R^n$ so that the diagram



is commutative. Here the notation $\times 0$ stands for the map $b \longrightarrow (b, 0)$, p_1 denotes the projection into the first factor, and R^n denotes Euclidean n-space.