A Note on a Linear Automorphism of R^n with the Pseudo-Orbit Tracing Property

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1. Introduction

A. Morimoto proved in [1] (Proposition 1) that for any linear automorphism f of $\boldsymbol{R}^n$, f is hyperbolic if and only if f has the pseudo-orbit tracing property (P.O.T.P.). To show that if f is not hyperbolic then f does not have the P.O.T.P., for $\delta > 0$ he constructed the δ-pseudo orbit (δ-p.o.) for which there are no tracing points. But the sequence of points that he constructed is not δ-p.o. for $n \geq 2$.

To supply this gap, we show in this paper that if f has the P.O.T.P. then f is hyperbolic.

2. Definition and lemmas

Let $f: X \to X$ be a homeomorphism of a metric space (X, d). We denote by $H(X)$ the group of all homeomorphisms of X.

DEFINITION. A sequence of points $\{x_n\}_{n \in \boldsymbol{Z}}$ is called a *δ-pseudo-orbit* (δ-p.o.) of f if $d(f(x_n), x_{n+1}) < \delta$ for $n \in \boldsymbol{Z}$. $\{x_n\}_{n \in \boldsymbol{Z}}$ is called to be *ε-traced* by $y \in X$ (with respect to f) if $d(f^n(y), x_n) < \varepsilon$ for $n \in \boldsymbol{Z}$. This y is called an *ε-tracing point*.

We say that f has the *pseudo-orbit tracing property* (P.O.T.P.) if for each $\varepsilon > 0$ there exists $\delta > 0$ such that any δ-p.o. of f can be ε-traced by some point $y \in X$.

We shall use the following lemmas given in [1] (or [2]).

LEMMA 1. *Let $h \in H(X)$ be a homeomorphism of X such that h and h^{-1} are both uniformly continuous. Take $f \in H(X)$ and put $g = h \circ f \circ h^{-1}$. Then f has the P.O.T.P. if and only if g has the P.O.T.P.*

LEMMA 2. *Let (X, d) and (X', d') be metric spaces, and let $f \in H(X)$ and $g \in H(X')$. The direct product $X \times X'$ is a metric space by the distance function $d''((x, x'), (y, y')) = Max\{d(x, y), d'(x', y')\}$ for $x, y \in X$ and $x', y' \in X'$. Put $(f \times g)(x, x') = (f(x), g(x'))$ for*