On Cone-Extreme Points in $\mathbf{R}^n$

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(Received September 22, 1986)

1. Introduction

Recently, the decision problems in $\boldsymbol{R}^n$ ordered by a convex cone have been investigated by many authors (cf. [1], [2], [4] and [7]). In [7], Yu used the nonpositive orthant $\boldsymbol{R}^n_-$ as a convex cone C and defined the cone extreme points. Further, he introduced the concept of acute to the convex cone C and showed that this led some properties of cone extreme points. Hartley introduced also the concept of cone compactness and showed that this is sufficient to guarantee the existence of an efficient point in [1]. Moreover, in [6], Tanino and Sawaragi introduced the concepts of $\boldsymbol{R}^p_+$-boundedness and $\boldsymbol{R}^p_+$-closedness, and gave some properties to $\boldsymbol{R}^p_+$-bounded sets and $\boldsymbol{R}^p_+$-closed sets.

In this paper we give the concepts of cone compactness, cone boundedness and cone closedness and investigate the characterization of the set of all cone extreme points of a subset A under a given cone C, denoted by Ext $[A \mid C]$. And we study the following:

(i) Ext $[A \mid C] \neq \phi$,

(ii) $A \subset \text{Ext}\,[A \mid C] + C$,

and

(iii) compactness or cone compactness of Ext $[A \mid C]$.

This paper is organized in the following way. In Section 2, we discuss the various properties of acute convex cones and Ext $[A \mid C]$. In Section 3, we study (i), (ii) and (iii) under C–compactness of a set A. In Section 4, we show them under C–boundedness and C–closedness of a set A. In addition, we investigate the relations among cone compactness, cone boundedness, and cone closedness.

2. Preliminaries and Cone Extreme Points

For a set $A \subset \boldsymbol{R}^n$, its closure, interior, and relative interior are denoted by cl$A$, int$A$, and ri$A$, respectively.