

# On an Optimal Multistrategy and a Weak Optimal Multistrategy of a Markov Game

By

LIU Zhaohua and Kensuke TANAKA

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## 1. Introduction

In the dynamic game theory, the multiperson game model with a discount factor on infinite horizon has been studied by many authors. The literature in this area is mostly concerned with the noncooperative equilibrium point. Such an equilibrium point gives the individual stability to each player, but it does not guarantee the collective stability. Actually, in many cases, the players may find a multistrategy which will yield a smaller total expected discounted loss if they cooperate. So, in [9], we proposed to find the D-solution which is analogous to the domination structure for a multiobjective decision problem.

In the paper, we introduce the distance from the total expected discounted loss constructed by all players to some given point as their collective loss function. All players cooperate in choosing a multistrategy to minimize this distance. But, in general, it would seem difficult to us to find directly such an optimal multistrategy. So, using some weighting factor vector, we modify our game system to a new one with the loss function weighting by this vector. Then, we develop the theory to find an optimal multistrategy, which is called a weak optimal multistrategy in the paper, in the modified game system. Moreover, we discuss the relation between an optimal multistrategy and a weak optimal multistrategy in the case which there exists an optimal one in the original game system. Finally, we show that a weak optimal stationary multistrategy in Theorem 1 is a D-solution under a domination structure determined by some convex cone  $D$ .

This paper is organized in the following way. In Section 2, we give a standard formulation for a cooperative  $m$ -person discounted Markov game. In Section 3, we give the necessary lemmas and definitions. In Section 4, we show the existence of a weak optimal multistrategy and discuss the relation between an optimal multistrategy and a weak optimal multistrategy. Finally, we show that a weak optimal multistrategy is a D-solution.